Energy Packet Networks with Finite Capacity Energy Queues

Sébastien Samain
INRIA Paris and DI ENS, CNRS, PSL Research University
Paris, France

Josu Doncel
University of the Basque Country, UPV/EHU
Leioa, Spain

Ana Busic
INRIA Paris and DI ENS, CNRS, PSL Research University
Paris, France

Jean-Michel Fourneau
Univ. Paris-Saclay, DAVID
Versailles, France

ABSTRACT

Energy Packet Network (EPN) consists of a queueing network formed by n blocks, where each of them is formed by one data queue, that handles the workload, and one energy queue, that handles packets of energy.

We study an EPN model where the energy packets start the transfer. In this model, energy packets are sent to the data queue of the same block. An energy packet routes one workload packet to the next block if the data queue is not empty, and it is lost otherwise.

We assume that the energy queues have a finite buffer size and if an energy packet arrives to the system when the buffer is full, jump-over blocking (JOB) is performed, and therefore with some probability it is sent to the data queue and it is lost otherwise.

We first provide a value of this probability such that the steady-state probability distribution of packets in the queues admits a product form solution. Moreover, in the case of a single block, we show that the number of data packets in the system decreases as the JOB probability increases.

CCS CONCEPTS

• **Mathematics of computing → Markov processes**; **Queueing theory**.

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1 INTRODUCTION

In the era of Internet of Things, Information and Communications Technology systems are growing at a very fast rate and, as a consequence, the performance analysis of such a huge network is a very challenging problem. Moreover, the source of energy that feeds this network includes an increasing amount of different types of renewable energies. The volatility of this kind of energy sources introduces clearly uncertainty in the amount of energy that is available in the future and, therefore, increases the difficulty of the optimal design of current communication systems.

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Current technology allows energy to be harvested. This means that energy can be stored in batteries or other devices so as to be used later. As a consequence, many researchers in the field of Computer Science have considered recently models where the energy is harvested. An example is the Energy Packet Network (EPN) model. This model has been introduced by Gelenbe and his colleagues [9] as a particular case of G-networks (we discuss in the related work section the literature in this topic). It considers that energy is represented by packets of discrete units of energy (Joules for instance) and, since its source is intermittent, it is assumed that arrivals to the system are given according to a random process. Therefore, in the EPN model, two types of packets are considered: on the one hand, the data packets that model the workload and are stored in the data queues; and on the other hand, the energy packets that are stored in the energy queues.

In this article, we study the EPN model where the energy packets start the transfer. This means that the energy packets are sent to the data queue and if, upon arrival, there is no data packet in the data queue, the energy packet is lost. However, if there are data packets available when the energy packet arrives, one data packet is sent to the next station and the energy packet disappears. This model captures well the performance of a system where tasks can only be executed when there is energy to feed the system. Sensor nodes and data centers are examples of these systems.

In the performance analysis literature, assumptions are sometimes considered that allow to get analytical results, but that are unrealistic from the practical point of view. This is the case, in fact, for most of the EPN models where the energy packet initiates the transfer, where it is considered that the energy queues (batteries) have a buffer of infinite size. In this article, we relax this assumption and we consider that the energy queues have a finite buffer size. We further assume that, if an energy packet arrives when the energy packet is full, there is a jump-over blocking (JOB), which means that it is sent to the data queue with some probability and it is lost otherwise.

The main contributions of this article are summarized as follows:

- We analyze the stationary distribution of packets in the queues and we show that it admits a product form solution for a given value of the probability at which an energy packet is sent to the data queue in case of jump-over blocking, i.e., in case of the energy queue is full.

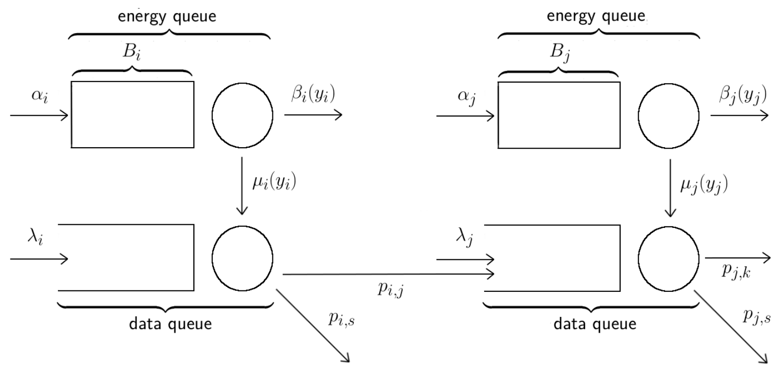

Figure 1: Block i and block j of the Energy Packet Network with energy queues with finite buffer size.

- For one block, we show that there exists a stochastic ordering according to the probability at which an energy packet is sent to the data queue in case of jump-over blocking. From this result, we conclude that the number of data packets of the system decreases with this probability.

We remark that we consider a very general model where some of the rates are a function of the number of energy packets in the system and, therefore, the results of this article cover a wide range of interesting cases such as energy queues that are single server and multiserver.

The remainder of the article is organized as follows. In Section 2, we put our work in the context of the existing literature. In Section 3, we describe the model we study in this article. We present the product form result in Section 4 and the stochastic ordering result in Section 5. Finally, we provide the main conclusions of our work in Section 6.

2 RELATED WORK

The Energy Packet Networks (EPNs) were introduced in [8–10] to represent the interaction of intermittent sources of energy from batteries or renewal energy such as solar or wind with Information Technology devices that consume energy. Since then, the EPN model has been successfully applied to analyse wireless sensors [12], mobile networks [11], computer systems design [13], data centers [4] and optimization of power distribution policies [14].

Most of the EPN models are particular cases of G-networks [5–7]. Therefore, the existence result of a product form of the steady-state distribution of packets in the queues of the latter model applies to the EPN model, which allows to investigate optimization problems as well as to design networks with energy harvesting. However, we observe that EPN models are not always related to G-networks, see for instance, the model in [1] where the authors use a diffusion approximation to solve the interactions between IT and energy.

All the EPN models that have been presented in the literature can be divided in two types depending on the initiator of the transfer. On the one hand, there are the models where the energy packets initiate the transfer (see for instance [13]). For this case, when the energy packets are sent to the data queue and are lost if there is no data packet. On the other hand, the data packets can start the transfer (see for example [3, 16]), in which case the data packets are sent to the energy queue and are routed to the next data queue if there are energy packets and lost otherwise. We note that, in both cases, when a successful transfer occurs, the energy packet is removed from the system, whereas the data packet is sent to the next station or leaves the system.

In this work, we study EPNs in a network where the energy queue has a finite buffer size, hence we extend the result of [15] where EPN models with a single node are considered. Since our model considers jump-over blocking in the energy queues, it is also clearly related to the set of queueing-theory papers where this technique is used, see for instance [17, 20] for its application to Jackson Networks. We refer to [2] for full details about product-form results of queueing-theoretical models with finite buffer size.

3 MODEL DESCRIPTION

In this section, we present the model of Energy Packet Network that we study. The network consists of n stations or blocks, each of them formed by a data queue and an energy queue. The arrivals to the data queue of block i follow a Poisson process with rate λ_i. The arrivals to the energy queue of the data queue of block i are also Poisson and its rate is denoted by α_i. A leakage of an energy packet occurs with exponential time. We consider that the rate at which leakage of an energy packet of block i occurs is a function of the number of energy packets in that block, i.e., if there are y_i energy packets in block i, the leakage rate is denoted by $\beta_i(y_i)$.

In our model, energy packets start the transfer. This means that an energy packet is sent to the data queue. We assume that the time required by an energy packet to reach the data queue is exponentially distributed with a rate that depends on the number of energy packets present in block i, that is, if there are y_i energy packets in the block i, this rate is denoted by $\mu_i(y_i)$. When an energy packet finds a data packet, the data packet is transmitted to the data queue of block j with probability $p_{i,j}$ and leaves the system with probability $p_{i,s}$. However, when the data queue is empty upon arrival of an energy packet, this energy is lost.

We consider that the energy queue of block i has a finite buffer size, which we denote by B_i. If an energy packet arrives to block

i when the energy queue of this block is full, jump-over blocking occurs. In other words, if the energy packet cannot be enqueued, it is either sent immediately to the data queue with jump-over blocking (JOB) probability q_i (where it transfers a data packet if the data queue is not empty and is lost otherwise) or lost with probability $1 - q_i$. We observe that, in both cases, the number of packets of the energy queue of block i does not change after the arrival of this energy packet, i.e., it remains full.

Two blocks of our EPN model are represented in Figure 1.

Throughout this article, we denote by $[\![a, b]\!]$ the set of all the integer values between a and b (where $a < b$). Besides, we denote by e_j the vector with the component in the position j equal to 1 and all other components equal to zero. Finally, we also denote by 1_A the indicator function of the event A.

4 PRODUCT FORM OF THE DISTRIBUTION OF PACKETS

In this section, we investigate the distribution of packets of the system described in the previous section. The main result of this section is that the distribution of packets in the queues has a product-form expression.

Let x be a vector that indicates the number of data packets in each block and y a vector of the number of energy packets in each block, i.e., $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ where x_i and y_i are respectively the number of data packets and of energy packets in block i. We denote by $\pi(x, y)$ the stationary distribution of packets in the queues. We now present the global balance equations:

$$
\begin{aligned}
&\pi(x,y)\Bigg(\sum_{i=1}^{n}\Big(\lambda_i + \alpha_i 1_{[y_i<B_i]} + \beta_i(y_i)1_{[y_i>0]} + \mu_i(y_i)1_{[y_i>0]}\\
&+\alpha_i q_i 1_{[y_i=B_i, x_i>0]}\Big)\Bigg) = \sum_{i=1}^{n}\Big(\lambda_i \pi(x-e_i, y)1_{[x_i>0]}\\
&+\alpha_i \pi(x, y-e_i)1_{[y_i>0]}\Big) + \sum_{i=1}^{n}\Big(\beta_i(y_i+1)\pi(x, y+e_i)1_{[y_i<B_i]}\Big) +\\
&\sum_{i=1}^{n}\Big(\mu_i(y_i+1)\pi(x, y+e_i)1_{[x_i=0, y_i<B_i]}\Big) +\\
&\sum_{i=1}^{n}\Big(\mu_i(y_i+1)p_{i,s}\pi(x+e_i, y+e_i)1_{[y_i<B_i]}\Big) +\\
&\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\mu_i(y_i+1)p_{i,j}\pi(x+e_i-e_j, y+e_i)1_{[y_i<B_i, x_j>0]}\Big) +\\
&\sum_{i=1}^{n}\Big(\alpha_i q_i p_{i,s}\pi(x+e_i, y)1_{[y_i=B_i]}\Big) +\\
&\sum_{i=1}^{n}\Bigg(\sum_{j=1}^{n}\alpha_i q_i p_{i,j}\pi(x+e_i-e_j, y)1_{[y_i=B_i, x_j>0]}\Bigg). \qquad (1)
\end{aligned}
$$

In the LHS of the above expression, we represent the total flow out from state (x, y). The RHS consists of 7 sums and it is formed by the total flow into (x,y). In the first sum, we represent the flow due to an arriving data packet and energy packet. In the second sum, we represent the flow due to the leakage of an energy packet. In the third sum, we represent the flow of an energy packet going to an empty data queue. In the next two sums, we represent the flow of an energy packet going to a non-empty data queue: in the first one the data packet served leaves the system and in the second one it is routed to another block. Finally, the last two sums show the flow into the state (x, y) in the case of the jump-over blocking, that is, when the energy queue is full. The penultimate sum represents the flow when an energy packet is sent to the data queue and finds a data packet, which leaves the system, whereas the last sum represents the case where the data packet does not leave the system and is sent to the next station.

In the following result, we show that when the rate of leakage and of travel time are equal (up to a constant factor), there exists a value of q_i such that the distribution of packets in the queues is given by a product form expression.

Theorem 4.1. *Let $f_i : [\![0, B_i]\!] \to [\![1, B_i]\!]$ and b_i and m_i a pair of constants (i.e., they do not depend on the state y_i) such that $\beta_i(y_i) = b_i f_i(y_i)$ and $\mu_i(y_i) = m_i f_i(y_i)$. We write $q_i = \frac{m_i}{m_i+b_i}$. Then*

$$\pi(x,y) = \left(\prod_{i=1}^{n} C_i(1-\rho_i)\rho_i^{x_i}\prod_{j=1}^{y_i}\gamma_i(j)\right) \qquad (2)$$

where for all $i \in [\![1, n]\!]$

$$\gamma_i(y_i) = \frac{\alpha_i}{(b_i+m_i)f_i(y_i)}, \quad y_i \in [\![1, B_i]\!], \qquad (3)$$

and

$$\rho_i = \frac{\lambda_i + \sum_{j=1}^{n}\alpha_j q_j \rho_j p_{j,i}}{\alpha_i q_i}, \qquad (4)$$

if $\rho_i < 1$ for all i.
Furthermore, the normalization constant is given by

$$C_i = \frac{1}{\sum_{j=0}^{B_i}\prod_{k=1}^{j}\gamma_i(k)}.$$

for all $i \in [\![1, n]\!]$.

Proof. We first check that π is a probability distribution as follows:

$$
\begin{aligned}
&\sum_{x\in[\![0,B]\!]^n}\sum_{y\in\mathbb{N}^n}\pi(x,y) = \sum_{x\in[\![0,B]\!]^n}\sum_{y\in\mathbb{N}^n}\left(\prod_{i=1}^{n}C_i(1-\rho_i)\rho_i^{x_i}\prod_{j=1}^{y_i}\gamma_i(j)\right)\\
&= \left(\sum_{x\in[\![0,B]\!]^n}\prod_{i=1}^{n}(1-\rho_i)\rho_i^{x_i}\right)\left(\sum_{y_i=0}^{B}\prod_{i=1}^{n}\left(C_i\prod_{j=1}^{y_i}\gamma_i(j)\right)\right)\\
&= \left(\prod_{i=1}^{n}\left(\sum_{x_i\in\mathbb{N}}(1-\rho_i)\rho_i^{x_i}\right)\right)\left(\prod_{i=1}^{n}\left(\sum_{y_i=0}^{B}\left(C_i\prod_{j=1}^{y_i}\gamma_i(j)\right)\right)\right)\\
&= \prod_{i=1}^{n}\left(\sum_{y_i=0}^{B}C_i\left(\prod_{j=1}^{y_i}\gamma_i(j)\right)\right) = \prod_{i=1}^{n}\left(C_i\left(\sum_{y_i=0}^{B}\prod_{j=1}^{y_i}\gamma_i(j)\right)\right)\\
&= \prod_{i=1}^{n}\left(\frac{\sum_{y_i=0}^{B}\prod_{j=1}^{y_i}\gamma_i(j)}{\sum_{y_i=0}^{B}\prod_{j=1}^{y_i}\gamma_i(j)}\right) = 1
\end{aligned}
$$

We now show that when $q_i = \frac{m_i}{m_i+b_i}$, the stationary distribution is equal to π given by the theorem. For this purpose, we show that it verifies the equation (1). We first notice that our product form is geometric and therefore the following properties follow directly:

- $\frac{\pi(x-e_i,y)}{\pi(x,y)} = \frac{1}{\rho_i}$,
- $\frac{\pi(x,y-e_i)}{\pi(x,y)} = \frac{1}{\gamma_i(y_i)}$,
- $\frac{\pi(x+e_i,y)}{\pi(x,y)} = \rho_i$,
- $\frac{\pi(x,y+e_i)}{\pi(x,y)} = \gamma_i(y_i+1)$,
- $\frac{\pi(x+e_i,y+e_i)}{\pi(x,y)} = \rho_i\gamma_i(y_i+1)$,
- $\frac{\pi(x+e_i-e_j,y+e_i)}{\pi(x,y)} = \frac{\rho_i\gamma_i(y_i+1)}{\rho_j}$,
- $\frac{\pi(x+e_i-e_j,y)}{\pi(x,y)} = \frac{\rho_i}{\rho_j}$.

We divide the expression in (1) by $\pi(x,y)$ and, using the list of identities above, we obtain the following equation:

$$\sum_{i=1}^{n}\left(\lambda_i + \alpha_i 1_{[y_i<B_i]} + \beta_i(y_i)1_{[y_i>0]}\right) =$$
$$+ \sum_{i=1}^{n}\left(\mu_i(y_i)1_{[y_i>0]} + \alpha_i q_i 1_{[y_i=B_i,x_i>0]}\right)$$
$$= \sum_{i=1}^{n}\left(\frac{\lambda_i}{\rho_i}1_{[x_i>0]} + \frac{\alpha_i}{\gamma_i(y_i)}1_{[y_i>0]} + \beta_i(y_i+1)\gamma_i(y_i+1)1_{[y_i<B_i]}\right)$$
$$+ \sum_{i=1}^{n}\left(\mu_i(y_i+1)\gamma_i(y_i+1)1_{[x_i=0,y_i<B_i]}\right)$$
$$+ \sum_{i=1}^{n}\left(\mu_i(y_i+1)p_{i,s}\rho_i\gamma_i(y_i+1)1_{[y_i<B_i]}\right)$$
$$+ \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\mu_i(y_i+1)p_{i,j}\frac{\rho_i\gamma_i(y_i+1)}{\rho_j}1_{[y_i<B_i,x_j>0]}\right)$$
$$+ \sum_{i=1}^{n}\left(\alpha_i q_i\rho_i p_{i,s}1_{[y_i=B_i]} + \sum_{j=1}^{n}\alpha_i q_i p_{i,j}\frac{\rho_i}{\rho_j}1_{[y_i=B_i,x_j>0]}\right).$$

Using the expression (3), the following properties follow directly:

- $\alpha_i q_i = \mu_i(y_i+1)\gamma_i(y_i+1)$
- $\frac{\alpha_i}{\gamma_i(y_i)}1_{[y_i>0]} = \beta_i(y_i)1_{[y_i>0]} + \mu_i(y_i)1_{[y_i>0]}$
- $\beta_i(y_i+1)\gamma_i(y_i+1) = \alpha_i(1-q_i)$

Therefore,

$$\sum_{i=1}^{n}\left(\lambda_i + \alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[y_i=B_i,x_i>0]}\right) \tag{5}$$
$$= \sum_{i=1}^{n}\left(\frac{\lambda_i}{\rho_i}1_{[x_i>0]} + (1-q_i)\alpha_i 1_{[y_i<B_i]}\right) \tag{6}$$
$$+ \sum_{i=1}^{n}\left(\alpha_i q_i 1_{[x_i=0,y_i<B_i]} + \alpha_i q_i\rho_i p_{i,s}\right) \tag{7}$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i q_i p_{i,j}\frac{\rho_i}{\rho_j}1_{[x_j>0]} \tag{8}$$

We now present the following equalities:

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i q_i p_{i,j}\frac{\rho_i}{\rho_j}1_{[x_j>0]} &= \sum_{j=1}^{n}\sum_{i=1}^{n}\alpha_i q_i p_{i,j}\frac{\rho_i}{\rho_j}1_{[x_j>0]}\\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_j q_j p_{j,i}\frac{\rho_j}{\rho_i}1_{[x_i>0]}\\ &= \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\alpha_j q_j p_{j,i}\frac{\rho_j}{\rho_i}\right)1_{[x_i>0]},\end{aligned} \tag{9}$$

where the first equality follows from a change in the order of the summations, the second equality follows from relabeling i and j and in the third equality follows from gathering terms.

Besides, we know from (4) that $\alpha_i q_i = \frac{\lambda_i + \sum_j \alpha_j q_j \rho_j p_{j,i}}{\rho_i}$. Therefore, using this and also the equality of (9), the expression (5)-(8) is equivalent to:

$$\sum_{i=1}^{n}\left(\lambda_i + \alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[y_i=B_i,x_i>0]}\right) \tag{10}$$
$$= \sum_{i=1}^{n}\left(\alpha_i q_i 1_{[x_i>0]} + (1-q_i)\alpha_i 1_{[y_i<B_i]}\right) \tag{11}$$
$$+ \sum_{i=1}^{n}\left(\alpha_i q_i 1_{[x_i=0,y_i<B_i]} + \alpha_i q_i\rho_i p_{i,s}\right) \tag{12}$$

We now show that the terms that are not multiplied by an indicator function are equal, i.e.

$$\sum_{i=1}^{n}\lambda_i = \sum_{i=1}^{n}\alpha_i q_i\rho_i p_{i,s}. \tag{13}$$

We prove this in the following way:

$$\begin{aligned}\sum_{i=1}^{n}\alpha_i q_i\rho_i p_{i,s}\gamma_i &= \sum_{i=1}^{n}\alpha_i q_i\rho_i\left(1-\sum_{j=1}^{n}p_{i,j}\right)\\ &= \sum_{i=1}^{n}\alpha_i q_i\rho_i - \sum_{i=1}^{n}\alpha_i q_i\rho_i\left(\sum_{j=1}^{n}p_{i,j}\right)\\ &= \sum_{i=1}^{n}\alpha_i q_i\rho_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i q_i\rho_i p_{i,j}\\ &= \sum_{i=1}^{n}\alpha_i q_i\rho_i - \sum_{j=1}^{n}\sum_{i=1}^{n}\alpha_i q_i\rho_i p_{i,j}\\ &= \sum_{i=1}^{n}\alpha_i q_i\rho_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_j q_j\rho_j p_{j,i}\\ &= \sum_{i=1}^{n}\lambda_i,\end{aligned}$$

where the last equation follows from (4).

We now prove that the equality for the rest of the terms is also verified in the following way:

$$\alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[y_i=B_i, x_i>0]}$$
$$= \alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[x_i>0]} - \alpha_i q_i 1_{[x_i>0, y_i<B_i]}$$
$$= \alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[x_i>0]} - \alpha_i q_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[x_i=0, y_i<B_i]}$$
$$= \alpha_i q_i 1_{[x_i>0]} + (1-q_i)\alpha_i 1_{[y_i<B_i]} + \alpha_i q_i 1_{[x_i=0, y_i<B_i]}$$

And, therefore, the desired result follows. □

We remark that our result covers a wide range of cases of interest. For instance, we can conclude the existence of a product-form when the energy queues are M/M/1/B_i queues since, for this case, we have that $\mu_i(y_i) = \mu_i$ and $\beta_i(y_i) = \beta_i$, which satisfy the condition of our theorem. Furthermore, the existence of the product form for energy queues that are M/M/B_i/B_i queues also follows from the above result since for that case $\mu_i(y_i) = y_i\mu_i$ and $\beta_i(y_i) = y_i\beta_i$.

We also remark that the value of ρ_i obtained in the result above does not depend on the value of the buffer size B_i and coincides with that of the corresponding EPN model with infinite capacity. The main reason for this is the way that the jump-over blocking is performed in our model. Besides, this property means that the model with infinite capacity and our model coincide in performance metrics of interest such as the mean number of data packets. However, we remark that, in our model, the stability of energy packets is not an issue, whereas in the infinite capacity packets it must be satisfied that $\gamma_i(y_i) < 1$.

Another interesting property we derive from the result above concerns the computation of the performance of the system. Indeed, since the steady-state distribution of packets has a product-form expression, one can calculate the mean number of data packets in the system as the sum of the mean number in each block of a network composed of independent queues but with the arrival rates equal to the effective arrival rates in our model. This is presented in the following result.

Corollary 4.2. *The mean number of data packets in the EPN model under consideration is given by*

$$\sum_{i=1}^{n} \frac{\rho_i}{1-\rho_i},$$

where ρ_i is given in (4).

5 STOCHASTIC ORDERING

In this section, we focus on a single block and we study the influence of the probability that an energy packet is sent to the data queue when the jump-over blocking occurs (i.e., when the energy queue is full). Since we consider a single block, we drop the subindex i of the parameters of the system in this part of the article.

We first define a partial order $\preceq_S$ on the state space S of our model. We say that $(x_1, y_1) \preceq_S (x_2, y_2)$ if $x_1 \leq x_2$ and $y_1 \geq y_2$. The intuitive idea of this ordering is that it is preferable to have (i) less data packets and (ii) more energy packets to be sent to the data queue and to route them to the next station. In other words, the energy packets play the role of the servers for the data queues.

We wish to compare the continuous time Markov chains (CTMC) corresponding to two single blocks with different values of JOB probabilities q according to the strong stochastic order; we first recall what is strong stochastic order for a pair of random variables:

Definition 5.1. Let $(S, \preceq_S)$ be a partially ordered space and X and Y two random variables on S. X is smaller than Y in a strong stochastic sense, noted $X \preceq_{st} Y$, if

$$E[f(X)] \leq E[f(Y)] \text{ for all increasing functions } f,$$

provided that the expectations exist.

Strong stochastic comparison of random variables on a partially ordered set can be characterized by means of increasing sets. A subset $\Gamma \subseteq S$ is called an increasing set if its indicator function 1_Γ is increasing. It follows that Γ is an increasing set if and only if $x \in \Gamma$ and $x \preceq_S y$ imply $y \in \Gamma$. The following characterization is often used as definition of st-order on a partially ordered space [18]. The proof can be found in [19].

Lemma 5.2. *$X \preceq_{st} Y$ if and only if $P(X \in \Gamma) \leq P(Y \in \Gamma)$, for all increasing sets $\Gamma \subseteq S$.*

We now define strong stochastic order for two processes:

Definition 5.3. Let $(S, \preceq_S)$ be a partially ordered space and X and Y two processes on S indexed by $\mathbb{R}_+$. X is smaller than Y in a strong stochastic sense, noted $X \preceq_{st} Y$ iff $\forall t \in \mathbb{R}_+, X_t \preceq_{st} Y_t$.

We now come to the main result of this section:

Theorem 5.4. *Consider an EPN network with a single block, when the energy queue is a M/M/1/B queue or a M/M/B/B queue; if X is a CTMC of this model with JOB probability q' and Y is a CTMC of this model with JOB probability $q \leq q'$, then*

$$X_t \preceq_{st} Y_t.$$

and its corollary:

Corollary 5.5. *The model with full rejection of the energy packet (i.e. $q = 0$) is greater (strong stochastic sense) than the model with jump-over blocking ($q > 0$).*

We now turn to the proof of theorem 5.4. We know that for CTMCs, the following characterization of strong stochastic order holds:

Theorem 5.6. *[18, Thm 5.3] Let $X = \{X_t\}_{t\geq 0}$ and $Y = \{Y_t\}_{t\geq 0}$ be two CTMC with infinitesimal generators Q and R. Then $X_t \preceq_{st} Y_t$, $\forall t \geq 0$ if and only if:*

- *$X_0 \preceq_{st} Y_0$,*
- *for all $u, v \in S$ such that $u \preceq_S v$ and for all increasing sets $\Gamma \subseteq S$ such that $u \in \Gamma$ or $v \notin \Gamma$ we have:*

$$\sum_{w\in\Gamma} Q(u, w) \leq \sum_{w\in\Gamma} R(v, w).$$

However, the conditions stated in the previous theorem may be difficult to check. Hence, we present a sufficient condition in corollary 5.10, which is easier to verify:

Definition 5.7. A CTMC $X = \{X_t\}_{t\geq 0}$ is monotone if for any two initial distributions μ and ν of X_0 such that $\mu \preceq_{st} \nu$ we have:

$$\forall t > 0, X_t^{\mu} \preceq_{st} X_t^{\nu},$$

where X_t^{μ} denotes that the initial distribution of X_0 is μ.

Theorem 5.8. *[18, Thm 5.2] A CTMC $X = \{X_t\}_{t\geq 0}$ with an infinitesimal generator Q is st-monotone if and only if for all $u, v \in S$ such that $u \preceq_S v$ and for all increasing sets $\Gamma \subseteq S$ such that $u \in \Gamma$ or $v \notin \Gamma$:*

$$\sum_{w\in\Gamma} Q(u,w) \leq \sum_{z\in\Gamma} Q(v,w).$$

Definition 5.9. Let Q and R be two generators. Then $Q \preceq_{st} R$ if for any $u \in S$ and for all increasing sets $\Gamma \subseteq S$ we have:

$$\sum_{w\in\Gamma} Q(u,w) \leq \sum_{w\in\Gamma} R(u,w).$$

We have the following sufficient condition:

Corollary 5.10. *Let $X = \{X_t\}_{t\geq 0}$ and $Y = \{Y_t\}_{t\geq 0}$ be two CTMC with infinitesimal generators Q and R. Then $X_t \preceq_{st} Y_t,\ \forall t \geq 0$ if*

- *$X_0 \preceq_{st} Y_0$,*
- *there is an st-monotone generator A such that*

$$Q \preceq_{st} A \preceq_{st} R.$$

We prove that this corollary holds for one-block EPN by using the following event representation (below, $z = (x,y)$):

- $a1$: *arrival of an energy packet of type 1* (jump-over blocking performed when the energy queue is full) with rate $\tau_{a1}(z) = q\alpha$;
- $a2$: *arrival of an energy packet of type 2* (jump-over blocking not performed when the energy queue is full) with rate $\tau_{a2}(z) = (1-q)\alpha$;
- d: *arrival of a data packet* with rate $\tau_d(z) = \lambda$;
- b: *leakage of an energy packet* with rate $\tau_b(z) = \beta(y)$;
- s: *service of a data packet, triggered by an energy packet* with rate $\tau_s(z) = \mu(y)$.

The reason behind splitting arrivals of energy packets into two types of events is purely to be able to describe the effect of the event by deterministic functions. To each event e, we associate a function $t_e : S \to S$ defined as follows:

- $a1$: $t_{a1}(x,y) = (x, y+1)1_{[y<B]} + ((x-1)^+, y)1_{[y=B]}$;
- $a2$: $t_{a2}(x,y) = (x, y+1)1_{[y<B]} + (x,y)1_{[y=B]}$;
- d: $t_d(x,y) = (x+1, y)$;
- b: $t_b(x,y) = (x, (y-1)^+)$;
- s: $t_s(x,y) = ((x-1)^+, y-1)1_{[y>0]} + (x,y)1_{[y=0]}$,

and a generator $Q_e = \Delta(\tau_e)(E(t_e) - I)$, with $\Delta(\tau_e)$ the diagonal matrix of rates and $E(t_e) = (1_{[t(z_1)=z_2]})_{(z_1,z_2)\in S\times S}$. Let $q \in [0,1]$ and $Q_{a,q} = Q_{a1} + Q_{a2}$ where the rate function for the events $a1$ and $a2$ are respectively αq and $\alpha(1-q)$. If Q is the generator of the CTMC associated to the one-block EPN with JOB probability q, then we have $Q = Q_d + Q_{a,q} + Q_b + Q_s$.

An event e is said to be st-monotone if its generator Q_e is st-monotone. If the generator Q of a CTMC can be written $Q = \sum_{e\in\mathcal{E}} Q_e$ such that every $e \in \mathcal{E}$ is st-monotone, then Q st-monotone [18]. Hence, we want to prove that every $e \in \{a1, a2, d, b, s\}$ is st-monotone. To this aim, we will use the following result from [18] that characterizes the st-monotonicity of an event:

Theorem 5.11 ([18, Thm 5.4]). *Let e be an event with destination $t : S \to S$ and rate $\tau : S \to \mathbb{R}^+$. Event e is st-monotone if and only if the following conditions are verified for all $z_1 = (x_1,y_1), z_2 = (x_2,y_2)$ such that $z_1 \preceq_S z_2$:*

1) *If $\tau(z_1)$ and $\tau(z_2)$ are nonzero, then at least one of the conditions must hold:*
   a) *$t(z_1) \preceq_S t(z_2)$,*
   b) *$z_1 \preceq_S t(z_2)$ and $t(z_1) \preceq_S z_2$.*
2) *If $\tau(z_1) < \tau(z_2)$, then $z_1 \preceq_S t(z_2)$.*
3) *If $\tau(z_1) > \tau(z_2)$, then $t(z_1) \preceq_S z_2$.*

In the two following lemmas, we show that all the events $e \in \{a1, a2, d, b, s\}$ are st-monotone for the cases where energy queue is a M/M/1/B queue or a M/M/B/B queue. We deal first with the M/M/1/B case:

Lemma 5.12. *Let $e \in \{a1, a2, b, d, s\}$ in a EPN with a single block, when the energy queue is an M/M/1/B queue. Then, e is st-monotone.*

Proof. All the rates are state-independent, i.e., $\tau_e(z)$ is a constant that only depends on $e \in \{a1, a2, b, d, s\}$. Therefore, conditions 2) and 3) of Theorem 5.11 are never verified and, as a consequence, to show that an event e is st-monotone, it is enough to show that condition 1) a) of Theorem 5.11 is satisfied.

To show this condition, for each event e, we partition the state space S into "types" of states such that t_e is a translation with respect to $\preceq_S$ on each "type set" (a subset of states of one given type), that is, if (a) is a type of states for e and $A_{(a)}$ is the subset of states of type (a), then $\forall z \in A_{(a)}, t_e(z) = z + v_{(a)}$, with $v_{(a)}$ a vector which depends only on the type (a); hence, on each of type sets, condition 1) a) will hold; moreover, when considering couples $(z_1,z_2) \in S^2$ such that $z_1 \preceq z_2$ and the pair (z_1,z_2) covers two different types (that is, z_1 is not of the same type as z_2), some cases will be forbidden; for instance, if a type (a) requires $z = (x,y)$ to be such that $x = 0$ and a type (b) requires $z = (x,y)$ to be such that $x > 0$, then we can have z_1 of type (a) and z_2 of type (b), but not the converse; similarly, if a type (a) requires $z = (x,y)$ to be such that $y = B$ and a type (b) requires $z = (x,y)$ to be such that $y < B$, then we can have z_1 of type (a) and z_2 of type (b), but not the converse; finally, if a type (a) requires $z = (x,y)$ to be such that $y > 0$ and a type (b) requires $z = (x,y)$ to be such that $y = 0$, then we can have z_1 of type (a) and z_2 of type (b), but not the converse.

In the following, it will be assumed that the states z, z_1 and z_2 can be written as (x,y), (x_1,y_1) and (x_2,y_2) respectively.

We first show that d is st-monotone. If $z_1 \preceq_S z_2$, we have $t_d(z_1) = (x_1+1, y_1) \preceq_S (x_2+1, y_2) = t_d(z_2)$. Hence, condition 1) a) holds.

We now show that b is st-monotone. For $z \in S$, we distinguish two types: (i) $y > 0$ and (ii) $y = 0$. Let $(z_1,z_2) \in S^2$ such that $z_1 \preceq_S z_2$. If z_1 and z_2 have same type then as t_b is a translation by $(0,-1)$ on $A_{(i)}$ and by $(0,0)$ on $A_{(ii)}$, condition 1) a) holds when z_1 and z_2 have the same type. If z_1 and z_2 cover types (i) and (ii), then z_1 is of type (i) and z_2 is of type (ii), and $t_b(z_1) = (x_1, y_1 - 1) \preceq_S (x_2, 0) = t_b(z_2)$. Hence, condition 1) a) holds.

We now show that s is st-monotone. For $z \in S$, we distinguish three types: (i) $y > 0, x > 0$, (ii) $y > 0, x = 0$ and (iii) $y = 0$. Let $(z_1, z_2) \in S^2$ such that $z_1 \preceq_S z_2$. As t_s is a translation by $(-1,-1)$ on $A_{(i)}$, by $(0,-1)$ on $A_{(ii)}$ and by $(0,0)$ on $A_{(iii)}$, condition 1) a) holds when z_1 and z_2 have the same type. If z_1 and z_2 cover types (i) and (ii), then z_1 is of type (ii), z_2 is of type (i) and we have $t_s(z_1) = (0, y_1 - 1) \preceq_S (x_2 - 1, y_2 - 1) = t_s(z_2)$. If z_1 and z_2 cover types (i) and (iii), then z_1 is of type (i), z_2 is of type (iii) and $t_s(z_1) = (x_1 - 1, y_1 - 1) \preceq_S (x_2 - 1, 0) \preceq_S (x_2, 0) = t_s(z_2)$. Finally, if z_1 and z_2 cover types (ii) and (iii) then z_1 is of type (ii) and z_2 is of type (iii), and $t_s(z_1) = (0, y_1 - 1) \preceq_S (x_2, 0) = t(z_2)$. Hence, condition 1) a) holds.

We now show that $a2$ is st-monotone. For $z \in S$, we distinguish two types: (i) $y < B$ and (ii) $y = B$. Let $(z_1, z_2) \in S^2$ such that $z_1 \preceq_S z_2$. As t_{a2} is a translation by $(0,1)$ on $A_{(i)}$ and by $(0,0)$ on $A_{(ii)}$, condition 1) a) holds when z_1 and z_2 have the same type. If z_1 and z_2 cover types (i) and (ii), then z_1 is of type (ii), z_2 is of type (i) and $t_{a2}(z1) = (x_1, B) \preceq_S (x_2, y_2 + 1) = t_{a2}(z_2)$. Hence, condition 1) a) holds.

We now show that $a1$ is st-monotone. For $z \in S$, we distinguish three types: (i) $y < B$, (ii) $y = B, x > 0$ and (iii) $y = B, x = 0$. Let $(z_1, z_2) \in S^2$ such that $z_1 \preceq_S z_2$; as t_{a_1} is a translation by $(0,1)$ on $A_{(i)}$, by $(-1,-1)$ on $A_{(ii)}$ and by $(0,0)$ on $A_{(iii)}$, condition 1) a) holds when z_1 and z_2 have the same type. If z_1 and z_2 cover types (i) and (ii), then z_1 is of type (ii), z_2 is of type (i) and $t_{a_1}(z_1) = (x_1 - 1, B) \preceq_S (x_1, B) \preceq_S (x_2, B) \preceq (x_2, y_2 + 1) = t_{a1}(z2)$. If z_1 and z_2 cover types (i) and (iii), then z_1 is of type (iii), z_2 is of type (i) and $t_{a1}(z_1) = (0, B) \preceq_S (x_2, y_2 + 1) = t_{a2}(z_2)$. Finally, if z_1 and z_2 cover types (ii) and type (iii), then z_1 is of type (iii), z_2 is of type (ii) and $t_{a1}(z_1) \preceq_S (0, B) \preceq_S (x_2 - 1, B) = t_{a1}(z_2)$. Hence, condition 1) a) holds. □

We now consider that the energy queues are M/M/B/B queues.

Lemma 5.13. *Let $e \in \{a1, a2, b, d, s\}$ in a EPN with a single block, when the energy queue is an M/M/B/B queue. Then, e is st-monotone.*

Proof. Using the same arguments as in the energy queue that is a M/M/1/B queue, we can easily show that events $a1$, $a2$ and d are st-monotone for this case as well.

The case of events b and s is different, because we have $\beta(y) = y\beta$ and $\mu(y) = y\mu$; hence, in addition to condition 1) a), we must also verify condition 3) for these events. Condition 1) a) as in the proof of lemma 5.9. Hence, we focus only on condition 3).

We show that the event b is st-monotone. For $z \in S$, we distinguish two types: $(i) y > 0$ and $(ii) y = 0$ Let $z_1 = (x_1, y_1), z_2 = (x_2, y_2) \in S$ such that $z_1 \preceq_S z_2$. If $y_1 = y_2$, then $\beta(y_1) = \beta(y_2)$ and we need not to verify condition 3). If $y_1 > y_2$, then $\beta(y_1) > \beta(y_2)$ and therefore, we need to verify that condition 3) of Theorem 5.11 is satisfied. In this case, z_1 must be of type (i) and z_2 can be either of type (i) or (ii). In both cases, we have $t_b(z_1) = (x_1, y_1 - 1) = (x_1, y_2) \preceq_S z_2$, and hence, condition 3) holds.

We now show that the event s is st-monotone. For $z \in S$, we distinguish three types:

- (i) states z for which $y > 0, x > 0$
- (ii) states z for which $y > 0, x = 0$
- (iii) states z for which $y = 0$

Let $z_1 = (x_1, y_1), z_2 = (x_2, y_2) \in S$ such that $z_1 \preceq_S z_2$. If $y_1 = y_2$, then $\mu(y_1) = \mu(y_2)$ and we need not to verify condition 3). If $y_1 > y_2$, then $\mu(y_1) > \mu(y_2)$ and therefore, we need to verify that condition 3) of Theorem 5.11 is satisfied. We distinguish the following cases:

- z_1 and z_2 are both of type (i): in this case, we have $t_s(z_1) = (x_1 - 1, y_2) \preceq_S (x_2, y_2) = z_2$, and hence condition 3) holds.
- z_1 and z_2 are both of type (ii): in this case, we have $t_s(z_1) = (0, y_1) \preceq_S (0, y_2) = z_2$, and hence condition 3) holds.
- z_1 and z_2 are of type (iii): in this case, the leakage rate is zero for both z_1 and z_2, and hence we do not need to prove condition 3).
- one of is of type (i) and the other is of type (ii): then z_1 is of type (ii) and z_2 is of type (i), and hence, we have $t_s(z_1) = (0, y_1) \preceq_S (0, y_2) \preceq_S z_2$, and hence condition 3) holds.
- one is of type (i) and the other is of type (iii): then z_1 is of type (i) and z_2 is of type (iii), and hence, we have $t_s(z_1) = (x_1 - 1, y_1 - 1) \preceq_S (x_2, 0) = z_2$, and hence condition 3) holds.
- one is of type (ii) and the other is of type (iii): then z_1 is of type (ii) and z_2 is of type (iii), and hence, $t_s(z_1) = (0, y_1 - 1) \preceq_S (x_2, 0) = z_2$, and hence condition 3) holds.

Hence, condition 3) holds for the event s.

□

We thus have proved, by summing the generators of the events, that the generator Q_q of the CTMC associated with the one-block EPN model with JOB probability q is st-monotone. We now want to show that when $(q, q') \in [0,1]^2$ are such that $q' \geq q$, then $Q_{q'} \preceq_{st} Q_q$. Again, our event representation is will help us, due to the following lemma:

Lemma 5.14. *Let $\mathcal{E}$ be a set of events and for any $e \in \mathcal{E}$, let Q_e and R_e be generators with $R_e \preceq_{st}$-monotone such that $Q_e \preceq_{st} R_e$, then $\sum_{e \in \mathcal{E}} Q_e \preceq_{st} \sum_{e \in \mathcal{E}} R_e$.*

As Q_e is independent of the JOB probability for any $e \in \{d, b, s\}$, and Q_e is st-monotone for any $e \in \{a1, a2, d, b, s\}$ when the energy queue is either M/M/1/B or M/M/B/B, we only need to show the following lemma:

Lemma 5.15. *If $q' > q$, then $Q_{a,q'} \preceq_{st} Q_{a,q}$.*

Proof. By definition 5.9, we need to show that for any $z = (x, y) \in S$ and any increasing set $\Gamma \subseteq S$, we have $\sum_{w \in \Gamma} Q_{a,q'}(z, w) \leq \sum_{w \in \Gamma} Q_{a,q}(z, w)$. To this aim, we distinguish two cases:

- $y < B$ or $z = (0, B)$: in this case, we have that $Q_{a,q}(z, .) = Q_{a,q'}(z, .)$, and we immediately have $\sum_{w \in \Gamma} Q_{a,q'}(z, w) \leq \sum_{w \in \Gamma} Q_{a,q}(z, w)$.
- $y = B$ and $x > 0$: in this case, as we have $t_{a1}(z) \preceq_S z$ and Γ is an increasing set, we only need to distinguish three cases:
  - $z \notin \Gamma$: in this case, $t_{a1}(z) \notin \Gamma$ and we necessarily have $\sum_{w \in \Gamma} Q_{a,q'}(z, w) = 0$ and $\sum_{w \in \Gamma} Q_{a,q}(z, w) = 0$.
  - $z \in \Gamma, t_{a1}(z) \notin \Gamma$: in this case, we have $\sum_{w \in \Gamma} Q_{a,q'}(z, w) = -\alpha q' \leq -\alpha q = \sum_{w \in \Gamma} Q_{a,q}(z, w)$.

– $z \in \Gamma, t_{a1}(z) \notin \Gamma$: in this case, we have $\sum_{w\in\Gamma} Q_{a,q'}(z,w) = Q_{a,q'}(z,z) + Q_{a,q'}(z, t_{a1}(z)) = -\alpha q' + \alpha q' = 0 = -\alpha q + \alpha q = Q_{a,q}(z,z) + Q_{a,q}(z, t_{a1}(z)) = \sum_{w\in\Gamma} Q_{a,q}(z,w)$.

Hence, in every case, we have $\sum_{w\in\Gamma} Q_{a,q'}(z,w) \leq \sum_{w\in\Gamma} Q_{a,q}(z,w)$; the lemma is thus proved □

Hence, we have proved theorem 5.4.

When we aim to extend the result of this section to a general network, we note that the problem is not easy. In fact, even though there are some properties that can be shown directly. For instance, the st-monotonicity of the events corresponding to the external arrivals of data packets can be shown using the same arguments as above. However, there is a difference when data packets can route from one block to the other. In that case, the routing events are not monotone for the partial order obtained as the product order of partial orders on single blocks as defined in this section. The order that one should define remains an open question.

6 CONCLUSION

In this paper, we study the EPN model where the capacity of the energy queues is finite and the energy packets start the transfer. This means that energy packets are sent to the data queue and, if the data queue is not empty, the data packet is sent to the next block (or leaves the system) and it is lost otherwise. When an energy packet arrives to block i where the energy queue is full, jump-over blocking occurs, i.e., either the energy packets is sent to the data queue, which occurs with probability q_i, or it is lost.

In a system with n blocks, we show that there exists a family of JOB probabilities $q_1, \ldots, q_n$ such that the steady-state probability distribution of packets in the queues has a product-form expression. The cases that are covered by our result include that the energy queue of block i is an $M/M/1/B_i$ queue and an $M/M/B_i/B_i$ queue.

For one block, we show that there exists a stochastic ordering according to the probability q. As a consequence, if we start two systems with different values of q from the same initial state, then the queue length at any time instant t (and at the steady-state) for the data queue is stochastically smaller in the st-sense for the system with higher value of q. At the same time, the number of energy packets at any time instant t (and in steady state) is bigger in the st-sense for the higher value of q. Note that this stochastic ordering does not use the product form property of the steady state distributions so it allows comparison of systems with values of q for which we do not have a product form result. In particular, the system in which all energy packets are lost when the energy queue is full is lower bounded by the system with the value of q that admits a product form.

In future work, we aim to generalize the stochastic ordering result to a network with more than one queue and extend our results to the EPN model where the data packets start the transfer.

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