Balancing energy consumption and losses with Energy Packet network models

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***Abstract*—We present models of Energy Packet Networks with a single and multiple class of customers. Such models were recently developed to study the interactions between IT capabilities and its energy consumption. These models are based on a discrete model of energy (the so called Energy Packets), some assumptions on the stochastic durations of energy production and consumption and the explicit description of the interactions between energy needed for computations or data transmission. These stochastic models have, under some classical assumptions on the arrivals processes, a steady-state distribution which has a product form. Such a closed form solution allows to optimize the design of the system and balance between losses, delay and energy.**

***Index Terms*—Energy, stochastic model, closed form solution.**

I. Introduction

Energy is one of the key problems for the terminals in fog computing systems. Energy harvesting is a promising solution for powering the IoT. It allows the devices (terminals, censors, actuators) to operate for a long period of time without providing new batteries. However, such a new alternative requires that we are able to study new network design problems related to the balancing of energy harvesting capacity and storage and the processing capacity. Energy is used to collect, store, process and transmit datas in the whole systems and the processes has to be balanced to proceed efficiently. Therefore we need new modeling approaches to help during this optimization process.

Energy Packet Networks (EPNs) is such a model. They were introduced recently by Gelenbe and his colleagues [11]–[14]. to explicitly represent the interactions between energy and IT in a network of computers or sensors. More precisely, it is used to model the flow of intermittent sources of energy like batteries and solar or wind based generators and their interactions with IT devices consuming energy like sensors, cpu, storage systems and networking elements.

The two key ideas of EPNs are the randomness of energy harvesting and the discretization of energy produced, consumed or stored in the model. Energy is modeled by packets of discrete units called Energy Packets (EPs). Each EP models a certain number of Joules. Since the EPs are produced by an intermittent source of energy (typically solar and wind), the flow of EPs is associated with some random processes. We also assume that the energy consumption by the devices are associated with some random process modeling the duration of the action. Typically if we model transmission of data, one EP is the energy needed to move a Data Packet between two adjacent nodes: one hop in the Data plane.

The EP can also be stored in a battery from which they can also leak after a random delay. Thus EPNs are associated with discrete state random processes evolving on continuous time. In the original EPN model presented for instance in [14], one represents the energy as EPs and the workload as Data Packets (DPs). To transmit a DP between two cells, one must use one EP. Hence, each cell in the network is associated with a server queue to store the DPs and a battery (the EP queue) to keep the energy. In this initial paper (i.e. [14]), the EPs are sent to the DP queue and triggers the customer movement between workload queues in the network. When an EP arrives at a DP queue which is not backlogged, the energy is lost. This is typically a network of queues with complex interactions between queues.

Since the seminal papers by Gelenbe on networks of queues with positive and negative customers [8], queues with triggers [9], or queues with batch deletions [10], the theory of networks of queues with customers and signals has been developed [3], [16], [17]. In this approach, at the completion of its service in a queue, the customer can become a signal which migrates to another queue. At the reception of the signal, the queue performs some action. It can also deny the effect of the signal. Access of failure of the action may be associated with a fixed or state dependent probability. Such an approach gives the opportunity to represent queues with complex synchronisations provoked y the signals.

Many EPN models can also be described by a network of queues with customers and signals. The importance of these models resides on the existence of a product form of the steady-state distribution of jobs in the queues. With such an exact closed form solution, it is possible to describe the optimization problems related to edge computing and to design the system to balance some utility functions like energy, losses or response time [5], [14], [15].

It is worthy to remark that EPN models are not always related to G-networks. They can be associated with various stochastic processes and several models or techniques. In [1] the authors use a diffusion approximation to solve the interactions between IT and energy. while the model in [13] is based on queues without signals. In [18] and [20], the authors use a stochastic process on Z to model the difference between DP and EP. Indeed, if the EP are in excess, the process state is negative. In these models the number of EP and DP are not explicitly represented. Even though the authors can obtain, under some probabilistic assumptions, the steady-state distribution of the system.

Note that EPN are not only a theoretical model. An independent approach has been presented in the electrical engineering literature under the name "power packet", see [23], [24]. In these papers, power packets are modeled as a pulse of current and are associated with a header and a protocol to control the routing using some hardware switching.

In this article, we present some results on EPN models with single and multiple classes of data packets. In section II, we describe three models of interactions with EP and DP with a single class of DP. These models differ by the interaction between the EPs and the DPs and the time needed for transfer. In a model the DP is the initiator of the transfer, that is, the arrival of a DP at the battery triggers the movement and, if a data packet does not find enough energy packets, it is lost. In the other models, the initiator is the EP. With the first model, the represent losses of data due to the lack of energy. With the second model, we represent the delay imposed by a energy.

Then in section III, we present a model for networks with multiple classes of data packets. The class of a DP determines the number of EPs required to be sent and the route it takes in the network. Furthermore, as usual with queues with multiple classes of customers, we consider several queuing disciplines: we consider that the DP queues operate under one of the following three scheduling disciplines: the First-Input-First-Output (FIFO), Last-Input-First-Output with Preemptive (LIFO-PR) and Processor Sharing (PS) queues as in [6]. Note that all these disciplines have already been considered for BCMP queues. We present the main results of the latter model in Section IV, that is, we show that the steady-state distribution of jobs in the queues has a product form provided that a stable solution of a fixed point problem exists.

In Section V, we focus on the stability of this model and also on the particular case of feed forward networks. More precisely, in Section V, we provide sufficient conditions for the stability of the EPN with an arbitrary topology. and, for feed forward networks, we show that the loads of all the queues can be characterized following the topological order of the network. Indeed, we observe that the fixed point problem is reduced to compute the roots of a polynomial.

We study in Section VI an example of a feed forward network where the the stability of the network can be easily characterized by computing the load of the DPs and of the EPs of each cell. We also investigated using an example the influence of the fragmentation of packets that required large amount of EPs to smaller packets with less energy requirements and we have seen that the loss rate in a given route increases with the energy requirements of the DPs and also that the mean number of customers decreases with the number of EPs required by the DPs to move to the following cell. Finally, we give the conclusion of this work in Section VII.

II. Single class model

We study a EPN with N cells in an open network, where each of the cells is formed by one server that stores DPs and one battery that stores EPs. EPs and DPs arrive following Poisson processes (with rate λ_i for DP, α_i for EP). Energy packets leak with an exponential distribution with rate β_i. To send a Data Packet to the

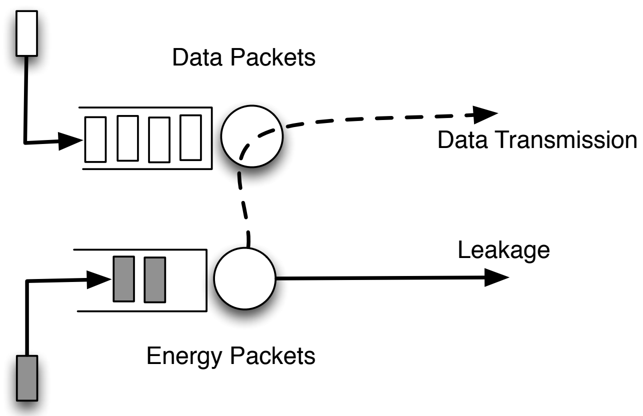

Fig. 1. First model of a cell of a single class EPN.

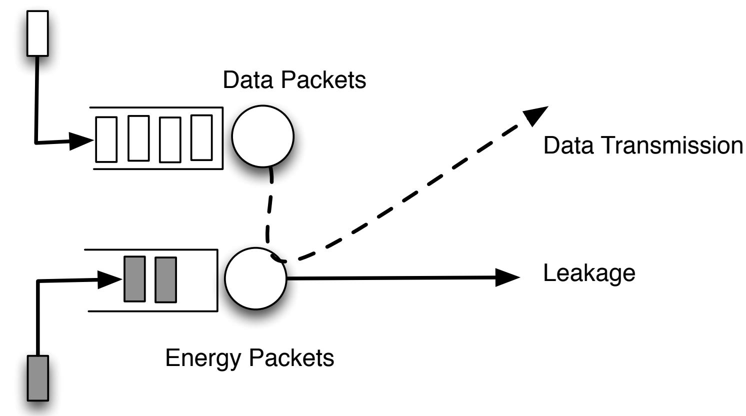

Fig. 2. Second model of a cell of a single class EPN.

next queue along the path, one Energy packet must be used the travel time between two adjacent nodes is exponentially distributed with rate μ_i. The routing matrix for DP is denoted as P. Note that Data Packets can be stored by the DP queue even if there is no energy available at the node.

The first question is the identity of the initiator of the data transfer. In the first model, we assume that the battery initiates the action. As a consequence, if the EP is sent to the DATA queue to carry the DP: the EP is lost is there is no DP available and the DP is delayed until the arrival of an EP. Thus this model represent the losses of energy (lack of DP at an arrival of EP) and delay for the DP (waiting for an EP to appear). This model is depicted in Fig. 1.

Theorem 1: Consider Markov chain $(X_i, Z_i)_{i=1..N}(t)$. X_i is the number of DP and Z_i the number of EP. Assume ergodicity. We assume that the EPs initiate the transfert and we do not consider the nodes on the boundary of the network which send DP to the outside. Let $\pi(X, Y)$ be the steady-state distribution:

$$\pi(X, Z) = \prod_{i=1}^{N} \rho_i^{X_i} \gamma_i^{Z_i} (1 - \rho_i)(1 - \gamma_i)$$

where ρ_i and γ_i are solutions of the flow equations:

$$\rho_i = \frac{\lambda_i + \sum_j \mu_j \rho_j \gamma_j P(j, i)}{\mu_i \gamma_i},$$

and

$$\gamma_i = \frac{\alpha_i}{\beta_i + \mu_i},$$

which must satisfy $\rho_i < 1$ and $\gamma_i < 1$ for all i.

It is worthy to remark that, unlike the Jackson networks, the flow equations linking ρ_i and γ_i is not linear. Thus we have to prove the existence of a solution and provide an algorithm to find the values of ρ_i, and γ_i.

Let us now consider an alternative model. Now we assume that the DP is sent to the ENERGY queue (i.e. the battery) to get the energy and move to the next hop. The DP is lost if there is no EP available and the EP is waiting until the arrival of a DP. Therefore more leakage may happen. Therefore this model may be used to study the losses of data due to the lack of energy and the losses of energy due to an excess of arrivals and leakage. Such a model is represented in Fig. 2.

We also have a product form result for this model.

Theorem 2: Consider Markov chain $(X_i, Z_i)_{i=1..N}(t)$. X_i is the number of DP and Z_i the number of EP, where the DPs initiate the transfert. Assume ergodicity of that chain. Again we do not consider here the nodes on the boundary to simplify the notation. Let $\pi(X, Y)$ be the steady-state distribution: if the flow equation

$$\rho_i = \frac{\lambda_i + \sum_j \mu_j \rho_j \gamma_j P(j, i)}{\mu_i},$$

and

$$\gamma_i = \frac{\alpha_i}{\beta_i + \mu_i \rho_i},$$

has a solution such that for all i, $\rho_i < 1$ and $\gamma_i < 1$, then

$$\pi(X, Z) = \prod_{i=1}^{N} \rho_i^{X_i} \gamma_i^{Z_i} (1 - \rho_i)(1 - \gamma_i)$$

Both theorems can be proved using global balance equation or the quasi-reversibility property.

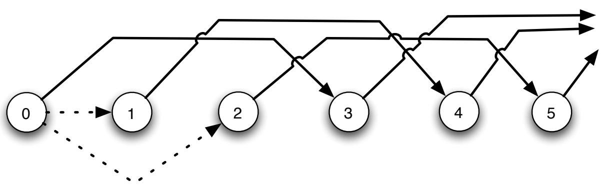

Fig. 3. Model of the arrival at a battery.

Both models can also be extended to deal with battery failures or ddos attacks agains the battery. We assume that the arrivals of failure follow a Poisson process with ϕ_i. The effect of the failure (or the attack) is to empty the battery instantaneously. But the battery is still alive and it can receive new EP. This action is sometimes denoted as a catastrophe signal in the literature on G-networks. We also have a closed form solution for both models. We only give one of them.

Theorem 3: Consider Markov chain $(X_i, Z_i)_{i=1..N}(t)$. X_i is the number of DP and Z_i the number of EP. We consider the model where the DPs initiate the transfert. Assume that the chain is ergodic. Let $\pi(X, Y)$ be the steady-state distribution: if the flow equation

$$\rho_i = \frac{\lambda_i + \sum_j \mu_j \rho_j \gamma_j P(j,i)}{\mu_i},$$

and

$$\gamma_i = \frac{\alpha_i}{\beta_i + \mu_i \rho_i + \phi_i (1-\gamma_i)^{-1}}$$

have a solution such that for all i, $\rho_i < 1$ and $\gamma_i < 1$, then

$$\pi(X, Z) = \prod_{i=1}^{N} \rho_i^{X_i} \gamma_i^{Z_i} (1-\rho_i)(1-\gamma_i)$$

The term $(1-\gamma_i)^{-1}$ at the denominator of γ_i makes the numerical resolution ore complex but an algorithm based on some equivalence of networks has been proved to deal with that topology (it can be considered as an instantaneous deletion loop) [7].

Let us now present a rather distinct approach [5]. First we assume that batch arrivals of EPs. The arrivals of new batteries still follow a Poisson process with rate ν_i but a fresh battery contains several Energy Packets. This batch size is constant. However when the battery is empty, we add an auxiliary arrival process of battery which are not completely filled (a kind of back pressure mechanism from the sensor to ask for more energy). This phenomenon is depicted in Fig. 3 with a batch size equal to 3.

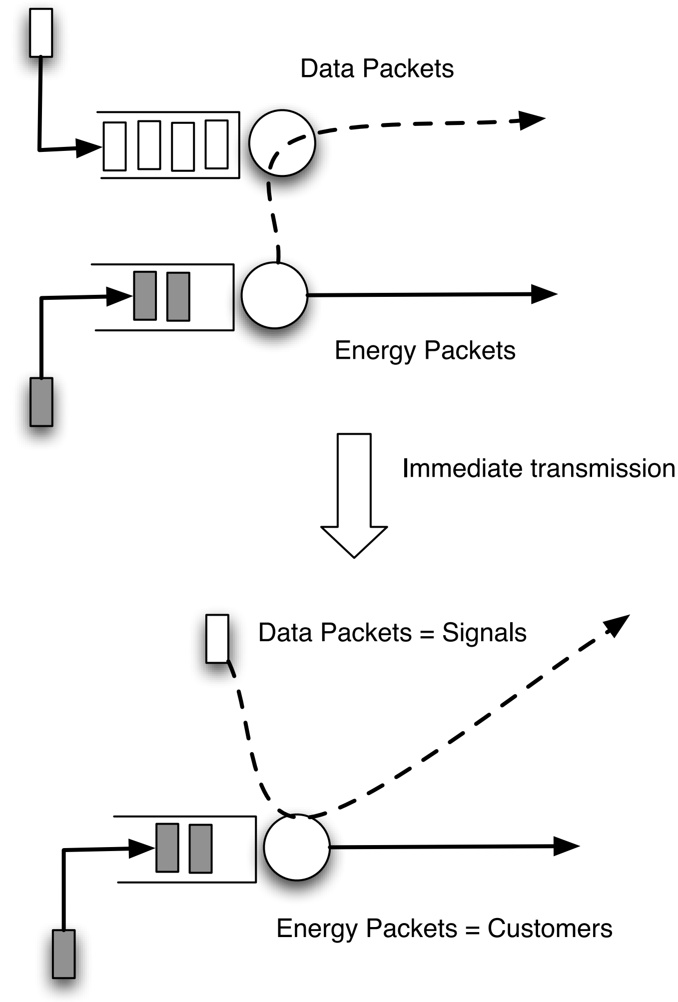

Fig. 4. Third model of a cell of a single class EPN. On the top, the classical EPN model consisting of 2 queues dedicated to EPs and DPs, respectively. The transmission of a DP is due to the movement of an EP. On the bottom, a new model: the EPs are stored in one queue modeling the battery and the DPs are modeled as instantaneous signals which decrease the battery energy level.

We still have the same interaction between Data Packets and Energy Packets But the travel time is now supposed to be instantaneous (i.e. typically short compared to the average time for an EP leakage or production). Thus the Data flows are represented by the footprints they make on the Energy queues (i.e. the batteries) and the model only represents the batteries (see Fig. 4).

One can find in [5] the product form result (which is not given here to avoid introducing to much notation).

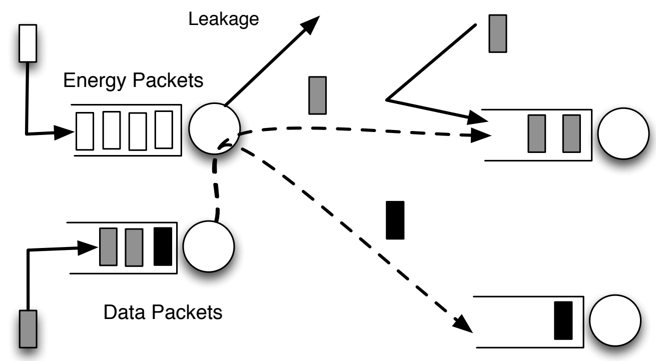

Fig. 5. A cell of a multiple class EPN. The two classes of data packets are represented in grey or black while the EP are in white. Grey DPs route to the upper DP queue while black DPs go to the lower DP queue. EP queues at the second stage are not represented to simplify the picture.

III. Multiple classes model

We now consider that there are K classes of DPs represented by different colors in Fig. 5. DPs arrive to each cell from outside the network following a Poisson distribution and we denote by $\lambda_i^{(k)}$ the arrival rate of class-k DPs to cell i. Likewise, EPs arrive to cell i following a Poisson distribution with rate α_i. We consider that there are energy leakages with exponential delay between losses of energy. Let $\beta_i > 0$ be the leakage rate of one EP in the battery of cell i. The service time of DPs is assumed to be exponentially distributed but it is now class-dependent. We denote by $\mu_i^{(k)} > 0$ the rate of the service time of class k jobs in the DP queue of cell i.

We consider a EPN where the DPs initiates the transfer. This means that, upon service in cell i, a DP of class k is sent to the battery of the same cell. The transfer is successful if there are at least $c_i^{(k)}$ EPs, in which case the DP is routed to the next cell and the $c_i^{(k)}$ EPs disappear. If the data packet finds less than $c_i^{(k)}$ EPs, then it is dropped. Furthermore we assume that there is some energy consumption to send the DP even if it fails. If there is a lack of energy before the end of the DP emission, the DP is lost and the energy consumed for the tentative is also lost. Thus, the battery gets empty.

Furthermore we assume that some nodes are on the boundary of the network and send DP to applications on the same machines or terminals. We assume that DPs do not consume energy when they leave the network because they stay on the same terminal but on the application level. The DPs move from one cell to another according to a fixed probability matrix, where $P_{i,j}^{(k)}$ represents the probability for a class k DP to move from the DP queue of cell i to the DP queue of cell j in case of a successful transfer, i.e., if there are more than $c_i^{(k)}$ EPs in the battery of cell i. Upon service in cell i, a DP packet of class k leaves the system with probability $d_i^{(k)}$. Hence, for all k and i, it follows that

$$d_i^{(k)} + \sum_{j=1}^{N} P_{i,j}^{(k)} = 1.$$

Like in [6], we consider three server types: First-Input-First-Output (FIFO), Processor Sharing (PS) and Preemptive Last-Input-First-Output (LIFO-PR). The FIFO discipline consists of giving serving to jobs in order of arrival. In the PS discipline all the jobs get a proportion of the processing capacity of the server and, if the number of jobs of class k jobs present in the system is x_k, the rate at which DPs of class k are served is given by $\frac{x_k}{\sum_{j=1}^{K} x_j}$. In the LIFO-PR discipline, the customers in the system constitute a stack, and the system serves always the customer that has been waiting for the shortest time.

A. State representation

We shall denote the state at time t of the queuing network by the vector $(\vec{X}, \vec{Z})$ where $\vec{X} = (x_1, ..., x_N)$ and x_i represents the state of DP service center of cell i and $\vec{Z} = (z_1(t), ..., z_N(t))$ and $z_i(t)$ is the number of EPs in the battery of cell i. The vector x_i depends on the queuing discipline of the DP queue of cell i and $||x_i||$ will be the total number of DPs in that queue.

For FIFO and LIFO-PR servers, the instantaneous value of the state x_i of server of cell i is represented by the vector $(r_{i,j})$ whose length is the number of customers in the queue and whose j-th element is the class index of the j-th customer in the queue. Furthermore, the customers are ordered according to the service order and it is always the customer at the head of the list which is in service. We denote by $r_{i,1}$ the class number of the customer in service and by $r_{i,\infty}$ the class number of the last customer in the queue.

For the PS servers, the instantaneous value of the state x_i is represented by the vector $(x_{i,k})$, where the k-th element represents the number of customers of class k at queue i.

IV. Main Results

Let $\Pi(\vec{X}, \vec{Z})$ denote the stationary probability distribution of the state of the network, if it exists. The following result states the product form solution of the network.

Theorem 4: Consider the EPN previously defined. If the system of non-linear equations:

$$\rho_i^{(k)} = \frac{\lambda_i^{(k)} + \sum_{j=1}^{N} \mu_j^{(k)} \rho_j^{(k)} P_{j,i}^{(k)} (\gamma_j)^{c_j^{(k)}}}{\mu_i^{(k)}}, \qquad (1)$$

and

$$\gamma_i = \frac{\alpha_i}{\beta_i + \sum_{l=1}^{K} \rho_i^{(l)} \mu_i^{(l)} \sum_{m=0}^{c_i^{(l)}-1} (\gamma_i)^m \sum_{j=1}^{N} P_{i,j}^{(l)}} \qquad (2)$$

has a solution such that: for each pair i, k, $0 < \rho_i^{(k)}$ and for each DP queue i, $\sum_{k=1}^{K} \rho_i^{(k)} < 1$ and EP queue i, $\gamma_i < 1$, then the stationary distribution of the network state is:

$$\Pi(\vec{X}, \vec{Z}) = G \prod_{i=1}^{N} g_i(x_i) \prod_{i=1}^{N} (1-\gamma_i)(\gamma_i)^{z_i}, \qquad (3)$$

where each $g_i(x_i)$ depends on the discipline of the service center of cell i. The $g_i(x_i)$ in Eq. 3 have the following form:

FIFO:
If the service center is FIFO, then

$$g_i(x_i) = \prod_{n=1}^{||x_i||} [\rho_i^{(r_{i,n})}] \qquad (4)$$

PS:
If the service center is PS, then

$$g_i(x_i) = ||x_i||! \prod_{k=1}^{K} \frac{(\rho_i^{(k)})^{x_{i,k}}}{x_{i,k}!} \qquad (5)$$

LIFO-PR:
If the service center is LIFO-PR, then

$$g_i(x_i) = \prod_{n=1}^{||x_i||} [\rho_i^{(r_{i,n})}] \qquad (6)$$

and G is the normalization constant. Since the network is open, G has a closed form expression which is given in Theorem 5.

The proof is based on simple algebraic manipulations of global balance equations, since it is not possible to use the "local balance" equations for customer classes at servers (see [4] for a proof).

As in BCMP [2] theorem, we can also compute the steady state distribution of the number of customers of each class in each queue. Let y_i be the vector whose elements are $(y_{i,k})$ the number of customers of class k in the server of cell i. Let $\vec{Y}$ be the vector of vectors (y_i).

Theorem 5: If the system of equations (1), and (2) has a solution then, the steady state distribution of the DPs $\Pi(\vec{Y})$ is given by

$$\Pi(\vec{Y}) = \prod_{i=1}^{N} h_i(y_i) \qquad (7)$$

where the marginal probabilities $h_i(y_i)$ have the following form :

$$h_i(y_i) = (1 - \sum_{k=1}^{K} \rho_i^{(k)}) |y_i|! \prod_{k=1}^{K} \frac{(q_{i,k})^{y_{i,k}}}{y_{i,k}!}. \qquad (8)$$

This proves that the normalization constant is $G = \prod_{i=1}^{N} (1 - \sum_{k=1}^{K} \rho_i^{(k)})$ and the desired result follows.

V. Stability, Feed Forward networks, and Algorithm

A. Stability

We now investigate the stability of the EPNs we study in this work. We know that the EPN is stable if and only if $\gamma_i < 1$ and $\sum_{k=1}^{K} \rho_i^{(k)} < 1$ for all cell i. Due to the lack of an explicit expression of the values of loads of EPs and DPs, providing necessary and sufficient conditions for the stability of this model seems to be an impossible task. However, in this section, we succeed in showing a sufficient condition for the stability of the EPN under consideration.

We first define the reduced network as the Jackson network that is formed by the servers only, i.e., without considering the batteries. We define by $q_i^{(k)}$ the load of DPs of class k of cell i in the reduced network. We know that the reduced network is stable if and only if for all i

$$\sum_{k=1}^{K} q_i^{(k)} = \sum_{k=1}^{K} \frac{\lambda_i^{(k)} + \sum_{j=1}^{N} \mu_j^{(k)} q_j^{(k)} P_{j,i}^{(k)}}{\mu_i^{(k)}} < 1.$$

We now focus on the batteries in isolation. For this case, we consider that the battery of cell i, which has arrival rate from outside α_i and leakage rate β_i, receives a traffic $\sum_{k=1}^{K} \lambda_i^{(k)}$ from the server of the same cell. We denote by p_i the load of the battery in isolation of cell i. We know that the battery in isolation of cell i is stable if and only if

$$p_i = \frac{\alpha_i}{\beta_i + \sum_{k=1}^{K} \lambda_i^{(k)}} < 1.$$

We now define the hyperstability of a EPN.

Definition 1: A EPN is hyperstable if its reduced network and all the batteries in isolation are stable.

In [4], we have shown that hyperstabilty implies stability.

Proposition 1: If a EPN is hyperstable, then it is stable.

From an algorithmic point of view, a simple iteration initialized with zero for ρ_i and γ_i converges in all the exemples we have studied. However we do not have formal proof of convergence of this numerical algorithm. To obtain stronger results one must consider more restricted topology, for instance the feed-forward networks.

B. Feed Forward Networks

We now focus on feed forward networks (i.e. networks associated with directed graphs without directed cycles). Thus a job that has been served by a given cell does not return to that cell almost surely. These graphs are associated with topological ordering of the nodes and this ordering is used to solve the equations as we now see.

We notice that, for a feed forward EPN, the load of class k jobs in the server of cell i is given by

$$\rho_i^{(k)} = \frac{\lambda_i^{(k)} + \sum_{j \in PRE(i)} \mu_j^{(k)} \rho_j^{(k)} P_{j,i}^{(k)} (\gamma_j)^{c_j^{(k)}}}{\mu_n^{(k)}}, \quad (9)$$

where $PRE(i)$ is the set of predecessor cells of i. As a result, one can compute the value of $\rho_i^{(k)}$ using only the values of the load of the servers and batteries of the predecessor cells.

The load of the battery of cell i in an feed forward network is given by

$$\gamma_i = \frac{\alpha_i}{\beta_i + \sum_{k=1}^{K} \mu_i^{(k)} \rho_i^{(k)} (1 - d_i^{(k)}) \sum_{m=0}^{c_i^{(k)}-1} (\gamma_i)^m}, \quad (10)$$

where the value of $\rho_i^{(k)}$ are obtained from Eq. 9. Hence, if the load of the batteries and of the servers of the predecessor cells is known, the computation of the load of the battery of cell i is reduced to calculate the roots of a polynomial of degree $\max\{c_i^{(1)}, \ldots, c_i^{(K)}\}$.

Instead of solving a fixed point problem, one compute the loads of the batteries and the servers according to the topological order of the nodes in the network: starting from the initial cells, i.e., those that do not have any predecessor, and continuing using the topological order of the network. This is described in Algorithm 1.

Algorithm 1 Algorithm to Compute the Loads of the Nodes in an Acyclic Graph

SET $PRE = \emptyset$.
FOR i s.t. $S^-(n) = \emptyset$.
    COMPUTE $\rho_i^{(k)}$ using (9) for all k.
    COMPUTE γ_i using (10).
    UPDATE $PRE = PRE \cup \{i\}$.
WHILE $\cup_{j \in PRE} S^+(j) \neq \emptyset$
    SET $V = \emptyset$.
    FOR $j \in PRE$
        FOR $i \in S^+(j)$
            COMPUTE $\rho_i^{(k)}$ using (9) for all k.
            COMPUTE $\gamma_{(k)}$ using (10).
            IF unstable solution of (10) or (9)
                RETURN -1.
            ELSE
                UPDATE $V = V \cup \{n\}$.
    SET $PRE = V$.
RETURN $\vec{\rho}_1, \vec{\rho}_2, \ldots, \vec{\rho}_K, \vec{\gamma}$

For these networks, we can invoke the results of Theorem 4 to conclude that (i) there exists a solution of the fixed point problem defined in Eq. 1 and Eq. 2 and (ii) if the fixed poing is stable, the distribution of jobs in this network has a product form. In this section, we show that the loads of the servers and of the batteries can be characterized for feed forward networks. After using Algorithm 1 we only need to verify that all the outputs of the algorithm are less than one.

VI. EXAMPLE

We now analyze an example of a feed forward EPN with eight cells and two types of DPs. First we aim to show that the values of the loads of the servers and the batteries can be easily obtained. The network under consideration is depicted in Figure 6. As the network is feed forward we use a numbering of the cells based on the topological ordering. Second we want to study the performance of the two routes from cell 1 to cell 8 in terms of loss rates and delays. Route 1 is dedicated to type 1 DPs (depicted in grey in the figure). It goes from cell 1 to cell 2, 3, and 4 and exits at cell 8. Route 2 conveys the type 2 DPs (in black). It begins at cell 1, passes through cells 5, 6 and 7 and ends at cell 8.

The DP server of cell 1 is the only one that receives traffic from outside the network. Therefore, we consider that $\lambda_i^{(1)} = \lambda_i^{(1)} = 0$ for all $i = 2, 3, \ldots, 8$. Besides, since the packets do not leave the system in the middle

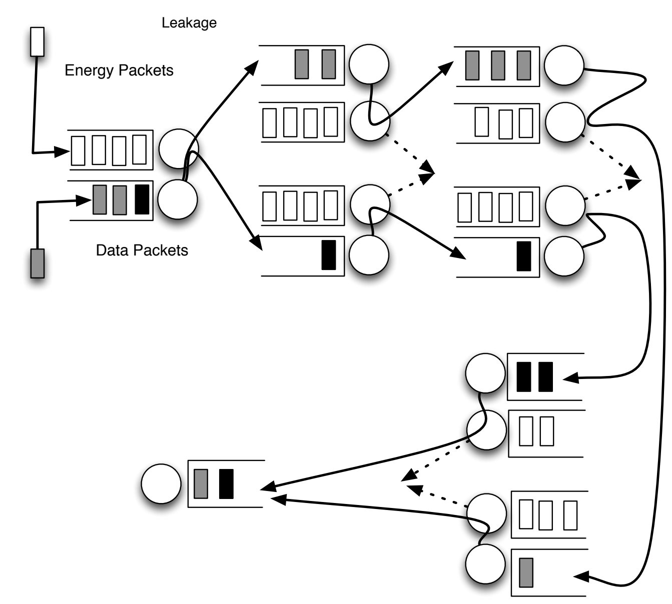

Fig. 6. Example of a directed feed forward EPN with 8 cells. The doted lines represent the energy leakage.

of the route, we have that $d_i^{(1)} = d_i^{(2)} = 0$ for all $i = 1, \ldots, 7$ and $d_8^{(1)} = d_8^{(2)} = 1$.

A. Computation of loads

We first compute the load of DPs and EPs in all cells of the network. We consider that the service rate of all the servers is equal to 1, except for the cell 1, where it is equal to 2, and the energy requirement of type 1 DPs in all the cells is equal to one whereas of type 2 DPs is two. Regarding the EP queues, the leakage rate of all the cells is 1 and the arrival rate to cells 1, 2, 3, 4 and 8 is 1.2 and to 5, 6 and 7 is 1.1.

Since this EPN is feed forward, we can obtain the values of the loads of the batteries and the servers so as to analyze its stability. We consider that $\lambda_1^{(1)} = 0.75$ and $\lambda_1^{(2)} = 0.5$Thus, we start with cell 1 and we obtain for the DPs that $\rho_1^{(1)} = 0.375$ and $\rho_1^{(2)} = 0.25$, whereas for the EP queue that $\gamma_1 = 0.5872$. We now focus on the next cells of Route 1, which are cells 2, 3 and 4 and only handle DPs of type 1. We obtain that, in cell 2, the load of type 1 DPs is 0.44 and the load of EPs is 0.8331. In cell 3, we have that the load of type 1 DPs is 0.3665 and the load of EPs is 0.8781 and in cell 4, we have that the load of type 1 DPs is 0.3218 and the load of EPs is 0.9078. We also compute the loads of the following cells of Route 2, i.e., cells 5, 6 and 7, where only DPs of type 2 are executed. We obtain that, in cell 5, the load of type 2 DPs is 0.1724 and the load of EPs is 0.9457. In cell 6, we have that the load of type 1 DPs is 0.1631 and the load of EPs is 0.9458 and in cell 7, the load of type 1 DPs is 0.1542 and the load of EPs is 0.9532. Finally, we compute the load of DPs in the last cell [1] and we obtain that the load of type 1 DPs is 0.2921 and the load of type 2 DPs is 0.1469. We conclude that the system is stable since in all the cells the load of the EPs is smaller than one and the sum of the loads of both types of classes is also smaller than one.

B. Fragmentation Analysis

Another interesting application of this model consists of studying how fragmentation of jobs affects in the performance of the system as we can divide large DP packets with a large value of $c_i^{(k)}$ into smaller DP packets requiring less energy.

We thus consider the topology of Figure 6 consisting of two types of DPs with equal load. Type 2 DPs are packets with an energy requirements equal to 2, i.e., $c_i^{(2)} = 2$ for all $i = 1, \ldots, 8$. The arrival rate of type 2 DPs is 0.4 and the service rate one. On the other hand, type 1 DPs are the fragmented packets and therefore the energy requirements of this type of DPs is equal to one. Besides, the arrival rate and the service rate of type 1 DPs is equal to 0.8 and 2, respectively. Hence, type 1 DPs represent packets with less energy requirements and type 2 DPs packets with higher energy requirements. We consider that the arrival rate to all the EP queues is equal to 1.2 (except for the EP queue of cell one, which is 2.5) and the leakage rate equal to 1.

The loss rate of a cell is the probability of a loss packet because in the EP queue of that cell there are less than the required EPs to be routed to the following cell when in all the predecessor nodes the transfer has been successful. For instance, the loss rate of cell 1 is $1 - \gamma_1 = 0.0955$ for type 1 DPs, whereas for type 2 DPs it is given by $1 - \gamma_1^2 = 0.17829$. We note that, since in cell 8 all the DPs leave the system once they have been served, then the loss rate in this cell is zero. In Table I we represent the loss rate of the remaining cells, that is, of cells 2, 3, 4, 5, 6 and 7.

We observe in Table I that the loss rate of cell 2, 3 and 4 is smaller than the loss rate of cell 5, 6 and 7, respectively. This means that the loss rate of Route 1 is less than the loss rate of Route 2. In other words, the

[1] Recall that all the jobs leave the system after being served in the cell 8 and, since they do so without energy requirements, we do not provide the value of the load of EPs in the last cell.

Loss Rate in Cell 2	Loss Rate in Cell 3	Loss Rate in Cell 4	Loss Rate in Cell 5	Loss Rate in Cell 6	Loss Rate in Cell 7
0.1994	0.0304	0.00795	0.4432	0.1128	0.0521

TABLE I
LOSS RATE IN EACH CELL OF THE EXAMPLE UNDER CONSIDERATION.

packets with smaller energy requirements have a smaller loss rate than the packets of higher energy requirements. Besides, in both routes, the loss rate decreases with the number of traversed cells. That is, in Route 1, the loss rate of cell 2 is higher than the loss rate of cell 3 and the loss rate of cell 3 is higher than the loss rate of cell 4. Likewise, in Route 2, the loss rate of cell 5 is higher than the loss rate of cell 6 and the loss rate of cell 6 is higher than the loss rate of cell 7.

Using the values obtained in Table I, we can also compute the total loss rate in each of the routes. For Route 1, it results that the total loss rate is 0.33325, whereas for Route 2 it is given by 0.78639. Therefore, this example shows that when the energy requirement of the DPs increases, so does the loss rate.

We now focus on the number of customers in the DP queues. We compute the mean number of customers of DPs in all the cells and the obtained values are represented in Table II. We observe that the mean number of customers in each route decreases with the number of cells that are traversed. For instance, the mean number of customers in cell 2 is 0.68896 and it is higher than the mean number of customers in cell 3, which is 0.12941; and the mean number of customers in cell 3 is higher than the mean number of customers in cell 4, which is 0.12751. The main reason for this is that there are no external arrivals in the considered example and, therefore, the load of the DPs of a given cell is smaller the that load of DPs of a predecessor one.

Another interesting conclusion of the results shown in Table II is that the total mean number of customers of type 1 DPs is higher than the mean number of customers of type 2 DPs. The explanation for this is the following: type 2 DPs require more energy to move from one cell to the following one, therefore, there are more DPs of type 2 that are lost and, as a result, the mean number of customer of type 1 DPs is higher that the mean number of customers of type 2 DPs.

Finally using Little's Law and due to the linearity of the expectation, one can compute the average end to end delay for Route 1 and Route 2. For Route 1, we have obtained that the mean delay is 6.99 seconds. However, for Route 2, we have that the mean delay is 13.93 seconds. This result shows the impact of the different service rates on the mean delay of two paths in a network.

In this example, we have studied how fragmentation affects on the performance of a queuing system. We have seen that the loss rate is higher when the energy requirements of the DPs is higher and also that the mean number of customer decreases with the number of EPs required to move to the following cell.

VII. CONCLUSIONS

The EPN approach is a new modeling paradigm which allows to model the interactions between energy and computation or data transfer.

In this paper, we have presented different EPN models where the distribution of packets in the queues is given by a closed form expression. We have considered systems with a single class of DPs and also multiple classes of DPs. For the latter model, we have also studied the effect of the fragmentation of DPs with high energy requirements to DPs with smaller energy requirements and we have concluded that: (i) the loss rate in a given route increases with the number of EPs required by the DPs to move the the next cell and (ii) the mean number of customers decreases with the number of EPs required by the DPs to move the the next cell. Interestingly, these results suggest that there is a trade-off between performance and loss rate when we vary the energy requirement of the DPs.

According to the results of this article, the distribution of packets in the network follows a product-form expression. Therefore, the results we have presented can be used to optimize the energy consumption, the losses or the delays. There is a large amount of possible extensions to these models. For instance, one can consider load-balancing techniques, as in [19], [21], [22], where idle queues can poll another queues. Moreover, another interesting extension of this model consists of studying optimal routes and efficient fragmentation of flows of data packets.

Mean Number of Customers	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8
Type 1 DPs	4.5	0.68896	0.12941	0.12752	0	0	0	0.14102
Type 2 DPs	4.5	0	0	0	0.58672	0.2053	0.13576	0.12133

TABLE II
MEAN WAITING TIME IN EACH CELL AND EACH CLASS OF THE EXAMPLE UNDER CONSIDERATION.

We hope that this short presentation will open avenues for new applications in the design of modern fog computing systems.

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