

# A Resource Sharing Game for the Freshness of Status Updates

[Extended Abstract]

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## 1. INTRODUCTION

Timely information is a crucial factor in a wide range of information, communication, and control systems. For instance, in autonomous driving systems, the state of the traffic and the location of the vehicles must be as recent as possible. The Age of Information is a relatively new metric that measures the freshness of the knowledge we have about the status of a remote system. More specifically, the Age of Information is the time elapsed since the generation of the last successfully received packet by the monitor containing information about the source. Since the seminal paper [2], in several models it has been observed that the policies that optimize performance metrics of interest in queueing theory do not necessarily minimize the Age of Information. Hence, there is a large number of queueing models that are open research problems regarding the Age of Information metric. We refer to [6] for a recent survey of the Age of Information.

We consider a system where diverse sources send status updates following a Poisson process through the same queue to a monitor. The service time of updates is exponentially distributed and the queue serves updates according to the Last-Generated-First-Served queue with preemption in service (indeed, the authors in [1] show the optimality of this policy for a single source). In order to keep the generation rates under control, we assume that the sources need to pay for sending status updates, being the payment of a source proportional to its load. Inspired by the Kelly Mechanism [3], we formulate a non-cooperative game where each source chooses the rate at which generates updates so as to minimize the sum of its Average Age of Information and its payment. Our goal is to measure the inefficiency due to the competition of different sources (i.e. the Nash equilibrium) with respect to the minimum total cost that can be achieved by means of the Price of Anarchy, defined as the worst possible ratio between the cost at Nash equilibrium and that of a global optimum [4].

We first focus on a system with homogeneous service rates. For this problem we fully characterize both the Nash equilibrium and the global optimum; moreover, we show that

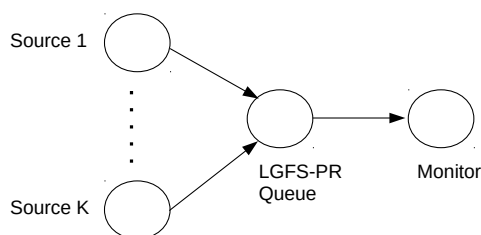


Figure 1: The system under study in this article.

the PoA of this case is  $2 - (1/K)$ , where  $K$  is the number of sources. In the case of heterogeneous service rates, we show that PoA is unbounded from above. One of the main conclusions of our work is that the inefficiency due to the competition of the sources is very small when the sources are homogeneous, but it can be arbitrarily large when the disparity between service rates increases.

## 2. MODEL DESCRIPTION

### 2.1 Game Formulation

We consider a system formed by  $K$  sources of information whose status is observed by a remote monitor. Each source generates status updates and sends them immediately through a transmission channel to the monitor. We assume that the transmission times from the sources to the transmission channel and from the transmission channel to the monitor are both negligible. Thus, the generation time of updates and the time at which updates arrive to the transmission channel coincide and similarly, the time at which updates end service and the delivery time of updates to the monitor coincide.

The transmission channel we consider is a Last-Generated-First-Served queue with preemption in service (LGFS-PR). Hence, when an update arrives to the queue it starts being served immediately, preempting the update currently in service if any.

The updates of source  $i$  are generated according to a Poisson process of rate  $\lambda_i$ . We assume that the service time of

updates of source  $i$  is exponentially distributed with rate  $\mu_i$ . Let  $\vec{\mu} = (\mu_1, \dots, \mu_K)$ ,  $\vec{\lambda} = (\lambda_1, \dots, \lambda_K)$ , and  $\lambda = \sum_i \lambda_i$ .

The Age of Information of source  $i$  at time  $t$  is defined as the difference between  $t$  and the generation time of the last update of source  $i$  that has been delivered to the monitor. We denote by  $\Delta_i(\vec{\lambda}, \vec{\mu})$  the Average Age of Information (AAoI) of source  $i$ .

We consider that the system planner penalizes a source if it generates updates very frequently. Specifically, we assume that each source must pay a cost for sending status updates that is proportional to its load, i.e., when the source  $i$  sends updates with rate  $\lambda_i$ , the payment of source  $i$  is  $c\lambda_i/\mu_i$ , where  $c > 0$ . The cost function of source  $i$  is defined as the sum of its AAoI and its payment, i.e.,

$$C_i(\vec{\lambda}, \vec{\mu}, c) = \Delta_i(\vec{\lambda}, \vec{\mu}) + c\lambda_i/\mu_i.$$

We define a game where each source is a player and aims to choose its generation rate so as to minimize its cost function, i.e.,

$$\min_{\lambda_i} C_i(\vec{\lambda}, \vec{\mu}, c). \quad (\text{GAME})$$

For homogeneous service rates,  $\Delta_i(\vec{\lambda}, \vec{\mu})$  is presented in (1), which was proven in [5], and we show that  $\Delta_i(\vec{\lambda}, \vec{\mu})$  for heterogeneous services rates is given by (6). It is worth mentioning that, in both cases,  $\Delta_i(\vec{\lambda}, \vec{\mu})$  is decreasing in  $\lambda_i$ .

Let  $\vec{\lambda}_{-i} = (\lambda_1, \dots, \lambda_{i-1}, \lambda_{i+1}, \dots, \lambda_K)$  and  $BR(\vec{\lambda}_{-i}, \vec{\mu}, c)$  be the best response of player  $i$  to  $\vec{\lambda}_{-i}$ , that is, the optimal generation rate of source  $i$  when the remaining sources have generation rates  $\vec{\lambda}_{-i}$ . A solution of (GAME) is called a *Nash Equilibrium* and we denote it as  $\vec{\lambda}^{ne} = (\lambda_1^{ne}, \dots, \lambda_K^{ne})$ . It is defined as a set of generation rates such that no source gets benefit from a unilateral deviation, i.e., for  $i = 1, \dots, K$

$$\lambda_i^{ne} \in BR(\vec{\lambda}_{-i}^{ne}, \vec{\mu}, c).$$

We note that the aforementioned relation between  $\Delta_i(\vec{\lambda}, \vec{\mu})$  and  $\lambda_i$  means that in a free-cost system (i.e. when  $c \rightarrow 0$ )  $\lambda_i^{ne} \rightarrow \infty, \forall i = 1, \dots, K$ .

## 2.2 Price of Anarchy

We now focus on the global optimization problem of this model. It consists of finding a set of generation rates such that the cost functions aggregated across sources is minimized, i.e.,

$$\min_{\{\lambda_1, \dots, \lambda_K\}} \sum_{i=1}^K C_i(\vec{\lambda}, \vec{\mu}, c). \quad (\text{GLOBAL-OPT})$$

A solution of (GLOBAL-OPT) is denoted by  $\vec{\lambda}^G$ .

The Price of Anarchy (PoA) is a widely studied performance metric to analyse the inefficiency of the Nash equilibria, and is defined as the worst possible ratio between the cost at Nash Equilibrium and that of the global optimum. For this model, it is given by

$$PoA = \sup_{\vec{\mu}, c} \frac{\sum_{i=1}^K C_i(\vec{\lambda}^{ne}, \vec{\mu}, c)}{\sum_{i=1}^K C_i(\vec{\lambda}^G, \vec{\mu}, c)}. \quad (\text{PoA})$$

In the following sections, we explore the PoA of this model. In Section 3 we consider the case of all sources having equal service rates and in Section 4 the case of different service rates.

## 3. HOMOGENEOUS SERVICE RATES

In this section, we study the PoA when  $\mu_i = \mu$  for all  $i$ . In [5], the authors show in Theorem 2a) that in case of identical service rates the AAoI is given by

$$\Delta_i(\vec{\lambda}, \mu, c) = \frac{\mu + \lambda}{\mu\lambda_i}. \quad (1)$$

Using (1), the cost function of source  $i$  is defined as

$$C_i(\vec{\lambda}, \mu, c) = \frac{\mu + \lambda}{\mu\lambda_i} + \frac{c\lambda_i}{\mu}. \quad (2)$$

We now provide the solution of (GAME) for this case.

**Lemma 1.** *When  $\mu_i = \mu$ , there exists a unique Nash Equilibrium and it is a symmetric equilibrium. Moreover, it is given by*

$$\lambda_i^{ne} = \frac{(K-1) + \sqrt{(K-1)^2 + 4\mu c}}{2c}, \quad i = 1, \dots, K. \quad (3)$$

We now characterize the solution of (GLOBAL-OPT) when the service rates are homogeneous.

**Lemma 2.** *When  $\mu_i = \mu$ , there exists a unique solution of the global optimization problem and it is given by*

$$\lambda_i^G = \sqrt{\mu/c}, \quad i = 1, \dots, K. \quad (4)$$

Note that  $\lambda_i^{ne}$  and  $\lambda_i^G$  tend to infinity as  $c \rightarrow 0$ , whereas they vanish when  $c \rightarrow \infty$ . In the next result, we show that the generation rates at the Nash Equilibrium are larger than the generation rates at the global optimization problem.

**Lemma 3.**  $\lambda_i^G < \lambda_i^{ne}, \forall i = 1, \dots, K$ .

We now focus on the PoA. When the service rates are homogeneous, using the definition (PoA), the cost function (2) and the optimal rates (3) and (4), it follows that

$$PoA = \sup_{\gamma} \frac{K + \sqrt{(K-1)^2 + 4\gamma}}{2\sqrt{\gamma} + K}, \quad (5)$$

where  $\gamma = \mu c$ . Since (5) is decreasing on  $\gamma$  it follows that the PoA is achieved when  $\gamma$  tends to 0, and the next result follows.

**Proposition 1.** *For the system with homogeneous service rates, the PoA is  $2 - \frac{1}{K}$ .*

## 4. HETEROGENEOUS SERVICE RATES

In this section we consider a model with different service rates. First we provide an expression for the AAoI of each source.

**Proposition 2.** *The AAoI of source  $i$  with heterogeneous service rates is*

$$\Delta_i(\vec{\lambda}, \vec{\mu}, c) = \frac{\mu_i + \lambda}{\mu_i\lambda_i}. \quad (6)$$

This proposition generalizes the result of Theorem 2a) in [5] and, as expected, both expressions (6) and (1) coincide

when  $\mu_i = \mu$ . We also note that (6) only depends on the service rate of source  $i$ . Indeed, since the updates in service are preempted upon arrival of a new update, the AAoI of source  $i$  does not depend on the service rates of other sources.

The cost function of source  $i$  in this scenario is therefore given by

$$C_i(\vec{\lambda}, \vec{\mu}, c) = \frac{\mu_i + \lambda}{\mu_i \lambda_i} + \frac{c \lambda_i}{\mu_i}. \quad (7)$$

In contrast to the homogeneous scenario, we have not succeeded in deriving a closed-form expression for the optimal generation rates with an arbitrary number of sources neither in the Nash Equilibrium nor in the global optimum. We thus set out to establish properties of the optimal solutions. For instance, in the following result we establish an ordering for the generation rates at the Nash Equilibrium.

**Lemma 4.** *If  $\mu_i \neq \mu_j \forall i \neq j$  then  $\mu_i > \mu_j \iff \lambda_i^{ne} > \lambda_j^{ne}$ .*

On the other hand, we now prove that there exists a particular cost, denoted by  $c^*$ , for which the generation rates at the solution of (GLOBAL-OPT) are equal for all sources.

**Lemma 5.** *Let  $c^* = K \left( \sum_{j=1}^K 1/\mu_j \right)$ . If  $\mu_i \neq \mu_j, \forall i \neq j$  then*

$$c = c^* \iff \lambda_i^G = \left( \sum_{j=1}^K 1/\mu_j \right)^{-1}, \forall i = 1, \dots, K.$$

Using the above results and assuming without loss of generality that  $\mu_1 > \dots > \mu_K$ , we can provide a lower bound of the ratio

$$\frac{\sum_{i=1}^K C_i(\vec{\lambda}^{ne}, \vec{\mu}, c^*)}{\sum_{i=1}^K C_i(\vec{\lambda}^G, \vec{\mu}, c^*)}, \quad (8)$$

that depends only on the generation rate at the Nash Equilibrium and the service rate of source  $K$ .

**Proposition 3.** *Assume  $\mu_1 > \dots > \mu_K$ . Then*

$$\frac{\sum_{i=1}^K C_i(\vec{\lambda}^{ne}, \vec{\mu}, c^*)}{\sum_{i=1}^K C_i(\vec{\lambda}^G, \vec{\mu}, c^*)} > \frac{2\lambda_K^{ne}}{3\mu_K} \quad (9)$$

Next we show that there exists a set of system parameters such that the ratio  $\lambda_K^{ne}/\mu_K$  can be indefinitely large.

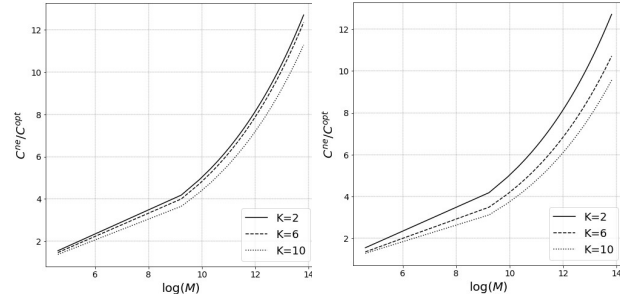
**Proposition 4.** *For any  $\theta \in \mathbb{R}^+$  there exist a set of service rates  $\{\mu_1, \dots, \mu_K\}$  such that, if  $c = c^*$ , then  $\lambda_K^{ne}/\mu_K > \theta$ .*

Finally, from (9) and last proposition we conclude that the ratio

$$\frac{\sum_{i=1}^K C_i(\vec{\lambda}^{ne}, \vec{\mu}, c^*)}{\sum_{i=1}^K C_i(\vec{\lambda}^G, \vec{\mu}, c^*)}$$

is unbounded from above, and consequently it follows the final result:

**Proposition 5.** *The PoA of the system with heterogeneous service rates is unbounded from above.*



**Figure 2: Ratio  $C^{NE}/C^{OPT}$  for Configuration 1 (left) and Configuration 2 (right)**

## 5. NUMERICAL EXPERIMENTS

This section is intended to illustrate numerically the results stated in Propositions 4 and 5. We define two different set of parameters for which  $\mu_1 > \dots > \mu_K$ : in Configuration 1 we take  $\mu_i = M - i + 1, i = 1, \dots, K - 1$  and  $\mu_K = 1$ , and in Configuration 2 we let the service rates be equally spaced along the interval  $[1, M]$ , that is,  $\mu_i = M - \frac{(M-1)(i-1)}{K-1}$ . Note that in both configurations  $M = \mu_1 - \mu_K$ . The cost is  $c = c^*$  as defined in Lemma 5. For both configurations, using a fixed-point algorithm we compute numerically the optimal generation rates, and we observed that for any value of  $\theta$ , there exists an  $M^*(\theta)$ , such that for all  $M \geq M^*(\theta)$ ,  $\lambda_K^{ne}/\mu_K > \theta$  (see Proposition 4), and furthermore, that  $M^*(\theta)$  is non-decreasing in  $\theta$ . This seems to indicate that as the range of the service rates grows, i.e.  $M$ , the ratio increases.

We illustrate this in Figure 2 where we depict the ratio (8) for different values of  $M$ . Figure 2 (left) corresponds to Configuration 1, and Figure 2 (right) to Configuration 2. In both cases we observe that, for any number of sources, as the value of  $M$  increases, so does the ratio. In future work we aim at establishing this property theoretically.

## 6. REFERENCES

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