A non-cooperative game between sensor nodes in an Energy Packet Networks model

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Joint work with I. Quintela (EHU) and O. Brun (LAAS-CNRS)

IV European Conference on Queueing Theory (ECQT) .
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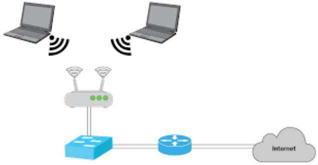
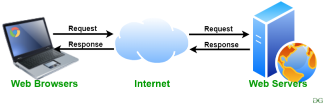
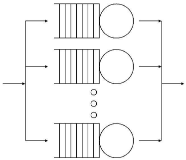
Outline

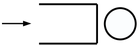
Motivation

Networks are often modeled as queueing systems









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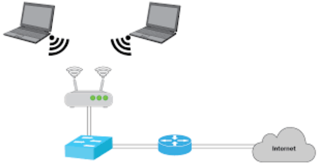

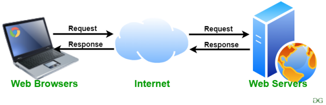

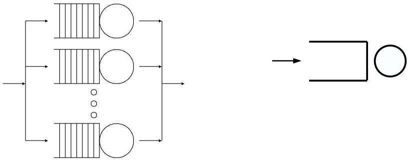

They require energy to work

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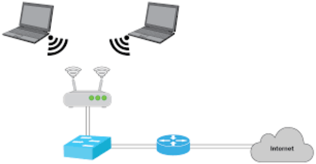

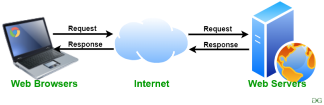

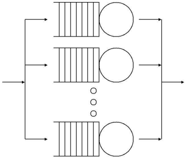

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Research considering energy in queueing systems is needed!

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Research based on energy is important

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Energy generated by renewal sources

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Research considering randomly generated energy in queueing systems is needed!

The Energy Packet Networks model

The Energy Packet Networks (EPN) model where the data starts the transfer

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One station of the EPN model

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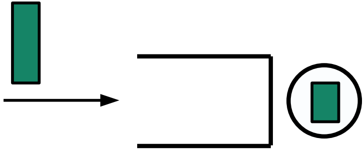

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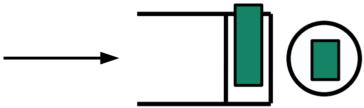

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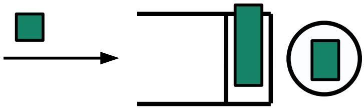

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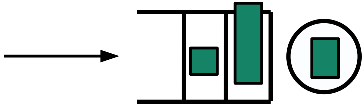

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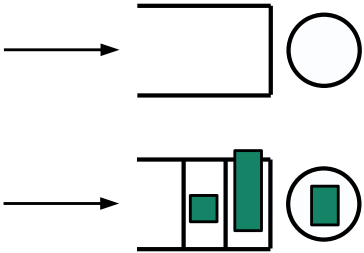

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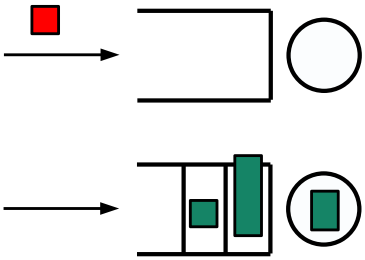

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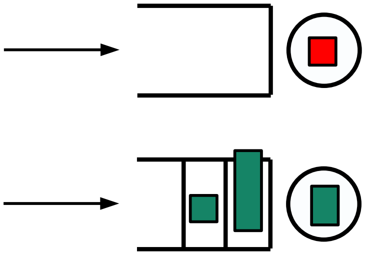

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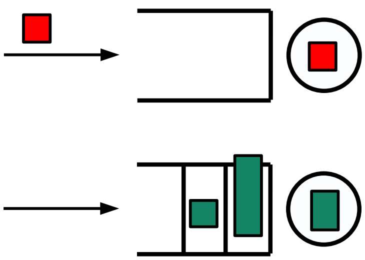

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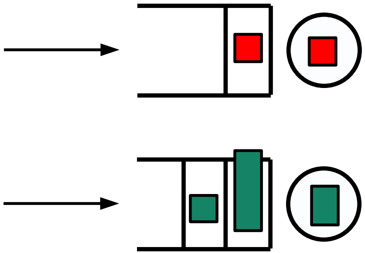

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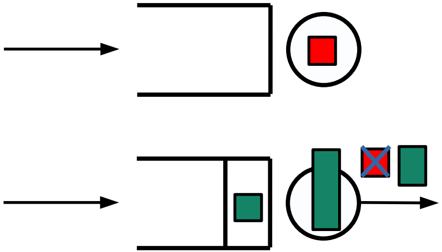

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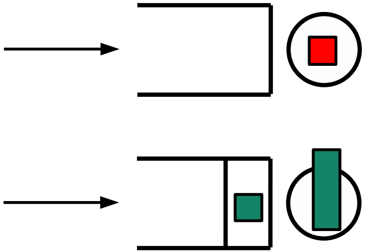

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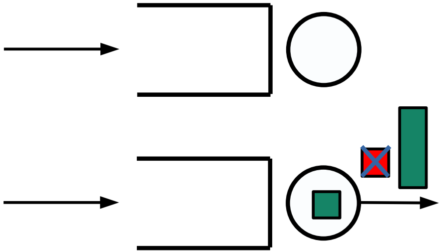

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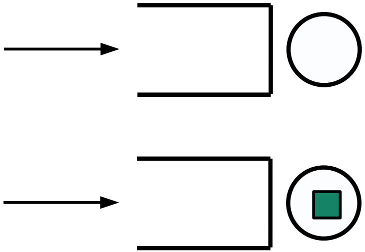

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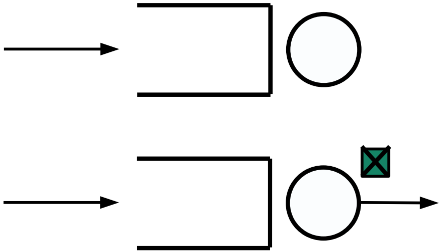

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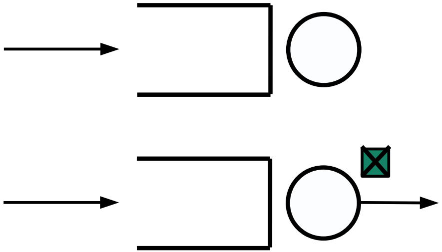

Unsuccessful transmission!

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Assume Poisson arrivals of data (with rate λ) and of energy (with rate α).
Data: exponential service times: μ
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Numerous works generalize or study variants this result

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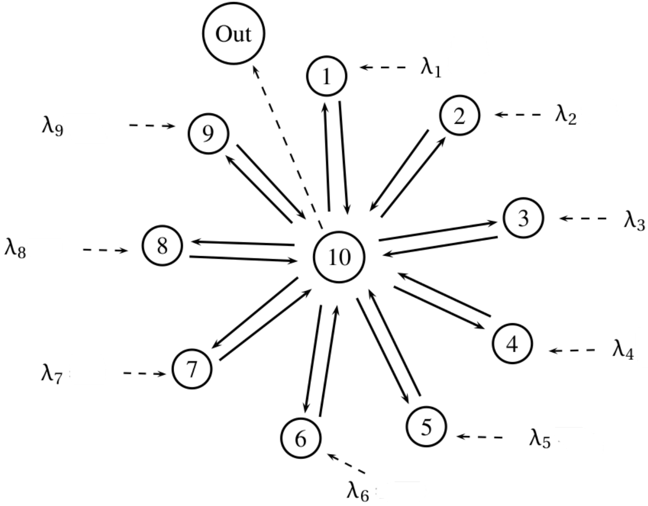

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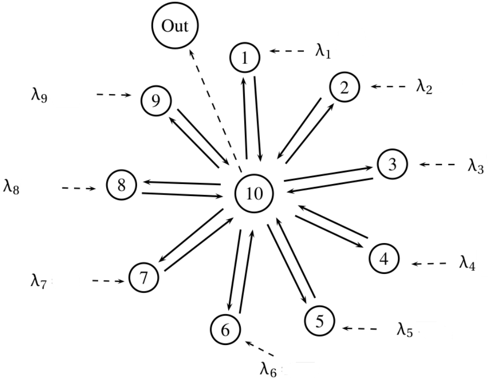

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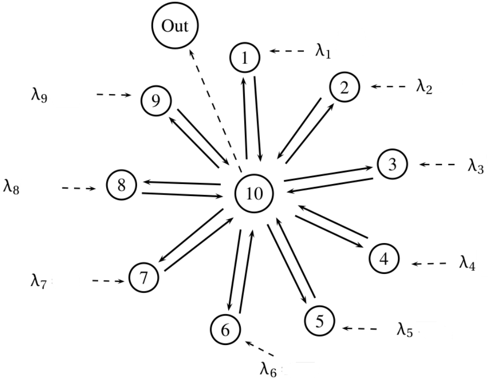

Sensor node i: λ_i, μ_i, β_i, α_i
Gateway with collisions

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Property [Bertsekas and Gallager 1992]

Assuming exponential service times with parameter μ, the probability of no collisions is

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$$\frac{e^{-\rho}}{1 + \rho}$$

Assume $\lambda_i < \mu_i$ for all sensor nodes

Game Formulation

Elements of the non-cooperative game

- Players: the sensor nodes

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$$F_i(\alpha_i, \alpha_{-i}) = \frac{\lambda_i \gamma_i \, e^{-\rho}}{1+\rho}.$$

The utility of sensor *i* depends on the strategy of all the sensors!

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The strategy of sensor node *i* when the strategy of the other players is fixed
Maximum over α_i of $F_i(\alpha_i, \alpha_{-i})$ when $\gamma_i < 1$.

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Maximum over α_i of $F_i(\alpha_i, \alpha_{-i})$ when $\gamma_i < 1$.

$$\underset{\alpha_i \in (0, \lambda_i + \beta_i)}{\arg\max} \; F_i(\alpha_i, \alpha_{-i})$$

Game Formulation

Nash equilibrium

The set of strategies $(\alpha_1^{NE}, \ldots, \alpha_K^{NE})$ is a NE when no player gets benefit from unilateral deviation.
For all $i = 1, \ldots, K$

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Global optimum

The set of strategies $(\alpha_1^*, \dots, \alpha_K^*)$ such that the total throughput is maximized

$$(\alpha_1^*, \dots, \alpha_K^*) = \underset{(\alpha_1, \dots, \alpha_K)}{\arg\max} \sum_{i=1}^{K} F_i(\alpha_i, \alpha_{-i})$$

such that $\alpha_i \in (0, \lambda_i + \beta_i)$ for all $i = 1, \dots, K$

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Unconstrained problem

$$\arg\max_{\alpha_i > 0} F_i(\alpha_i, \alpha_{-i}) \qquad \text{(UNCONST-BR)}$$

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Proposition

The unique maximum of (UNCONST-BR) is achieved at

$$\alpha_i^M = \frac{\mu(\lambda_i + \beta_i)}{2\lambda_i} \sqrt{1 + \rho_{-i}} \left(\sqrt{5 + \rho_{-i}} - \sqrt{1 + \rho_{-i}} \right),$$

where $\rho_{-i} = \rho - \frac{\lambda_i \gamma_i}{\mu}$.

Best Response to α_{-i}

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Constrained Problem: $\arg\max_{\alpha_i \in (0, \lambda_i + \beta_i)} F_i(\alpha_i, \alpha_{-i})$

When $\alpha_i^M < \lambda_i + \beta_i$, the best response of player *i* to α_{-i} is α_i^M

Nash equilibrium

Unconstrained problem

For $i = 1, \ldots, K$

$$\alpha_i^{NE} = \arg\max_{\alpha_i > 0} F_i(\alpha_i, \alpha_{-i}^{NE}) \qquad \text{(UNCONST-NE)}$$

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For $i = 1, \dots, K$

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Proposition

The solution to (UNCONST-NE) is denoted by $(\bar{\alpha}_1, \dots, \bar{\alpha}_K)$ and it is

$$\bar{\alpha}_i = \frac{\mu(\lambda_i + \beta_i)}{\lambda_i} \frac{K - 2 + \sqrt{K^2 + 4}}{2K}$$

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Observation: symmetric traffic to the gateway

For all i

$$\lambda_i \gamma_i = \frac{K - 2 + \sqrt{K^2 + 4}}{2K}$$

Nash equilibrium

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$$\frac{\bar{\alpha}_i}{\lambda_i + \beta_i} < 1 \iff \frac{K - 2 + \sqrt{K^2 + 4}}{2K} < \frac{\lambda_i}{\mu}$$

Nash equilibrium

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Proposition

When $\frac{K-2+\sqrt{K^2+4}}{2K} < \min_{i=1}^{K} \frac{\lambda_i}{\mu}$, $\alpha_i^{NE} = \bar{\alpha}_i$, for all $i = 1, \ldots, K$.

Global Optimum

Unconstrained problem

$$(\alpha_1^*, \ldots, \alpha_K^*) = \underset{(\alpha_1, \ldots, \alpha_K)}{\arg\max} \sum_{i=1}^{K} F_i(\alpha_i, \alpha_{-i}) \qquad \text{(UNCONST-GLO)}$$

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$F(\rho)$ single variable function!

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$F(\rho)$ single variable function!

$\arg\max F(\rho) = \frac{\sqrt{5}-1}{2}$.

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Unconstrained problem

$$(\alpha_1^*, \ldots, \alpha_K^*) = \underset{(\alpha_1,\ldots,\alpha_K)}{\arg\max} \sum_{i=1}^{K} F_i(\alpha_i, \alpha_{-i}) \qquad \text{(UNCONST-GLO)}$$

$$\sum_{i=1}^{K} F_i(\alpha_i, \alpha_{-i}) = \frac{\sum_{i=1}^{K} \lambda_i \gamma_i \, e^{-\rho}}{1+\rho} = \frac{\mu\rho \, e^{-\rho}}{1+\rho} = \mu F(\rho)$$

$F(\rho)$ single variable function!

$\arg\max F(\rho) = \frac{\sqrt{5}-1}{2}$.

Proposition

Any $(\alpha_1, \ldots, \alpha_K)$ such that $\rho = \frac{\sqrt{5}-1}{2}$ is a solution to (UNCONST-GLO).

Global Optimum

Constrained problem

When $\frac{1}{\mu}\sum_i \lambda_i > \frac{\sqrt{5}-1}{2}$, any $(\alpha_1, \ldots, \alpha_K)$ such that $\rho = \frac{\sqrt{5}-1}{2}$ is a global optimum.

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Goal: compare the global optimum and the Nash equilibrium

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By definition, $F\left(\rho^{NE}\right) \leq F\left(\frac{\sqrt{5}-1}{2}\right) \approx 0.206$.

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Work in progress: Numerical observation

For every K:

- the Nash equilibrium can be arbitrarily inefficient: $F(\rho^{NE}) \to 0$ when $\mu \to 0$
- the Nash equilibrium can be efficient: there exists a $\mu(K)$ such that

$$F\left(\frac{1}{\mu(K)}\frac{K-2+\sqrt{K^2+4}}{2}\right) = 0.206$$

Outline

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EPN model: queueing systems + energy

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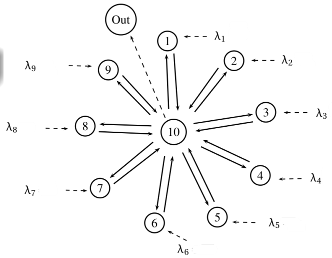
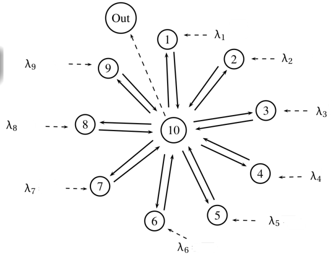


Conclusions and Future Work

EPN model: queueing systems + energy



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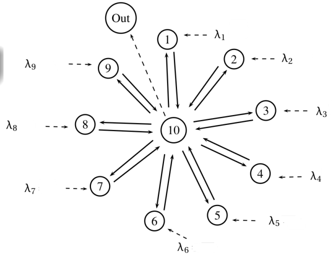

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Future Work

Efficiency analysis, other global optimum metrics or more realistic setting

That's all folks

Thanks for your attention

Questions?